12: Polyphase Filters

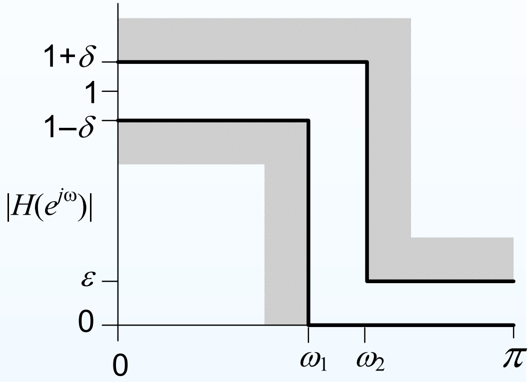
Heavy Lowpass filtering

Filter Specification:

Sample Rate: 20 kHz

Passband edge: 100 Hz ($\omega_1 = 0.03$)

Stopband edge: 300 Hz ($\omega_2 = 0.09$)



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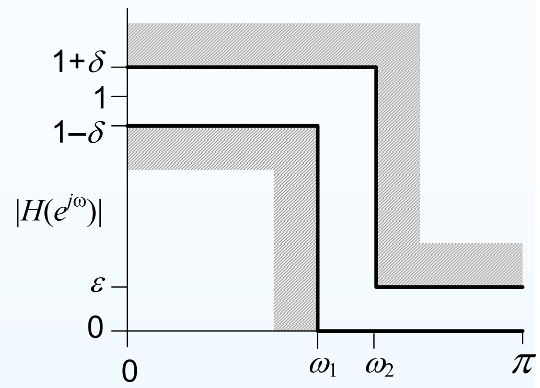
Passband ripple: ± 0.05 dB ($\delta = 0.006$)



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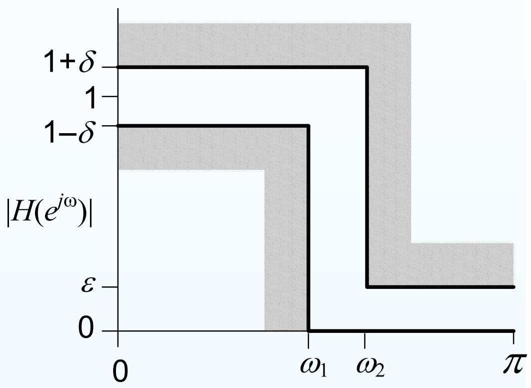

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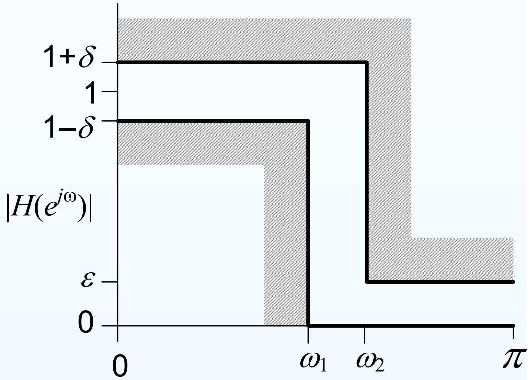

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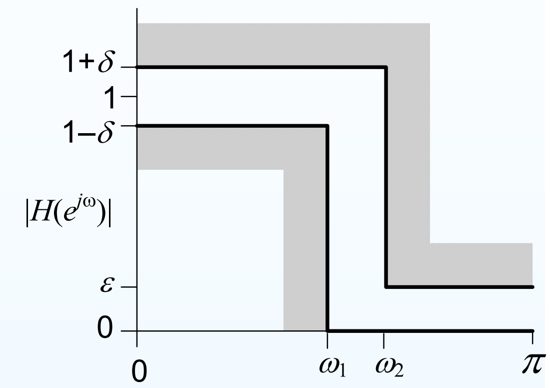

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Symmetric FIR Filter:

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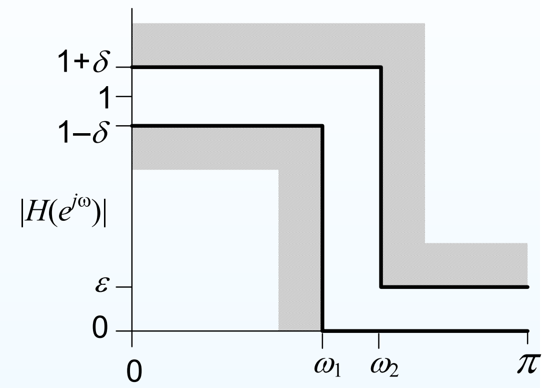

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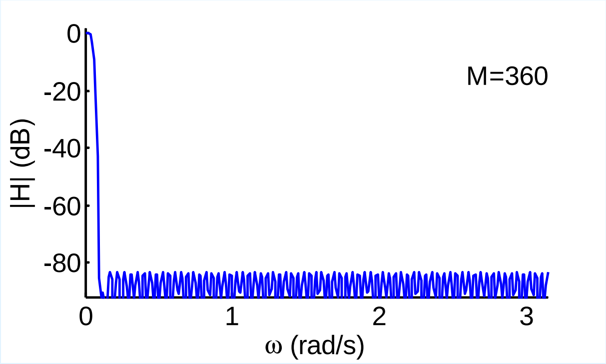
Symmetric FIR Filter:

Design with Remez-exchange algorithm

Order = 360



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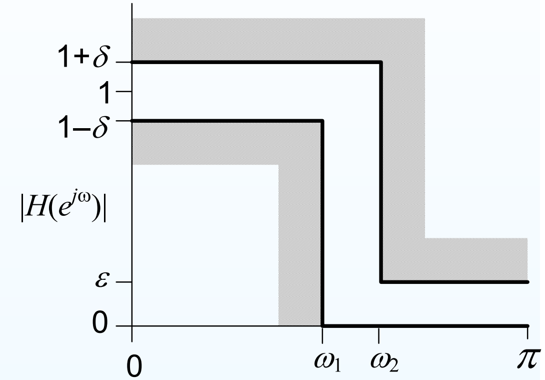

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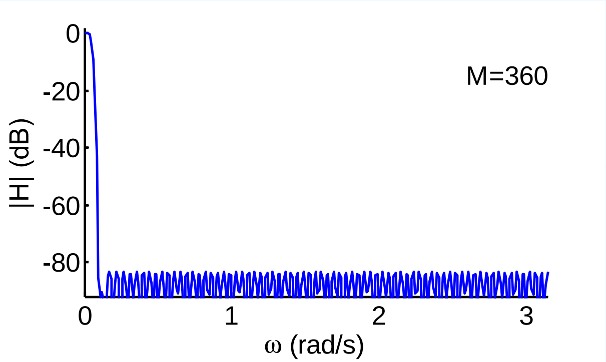
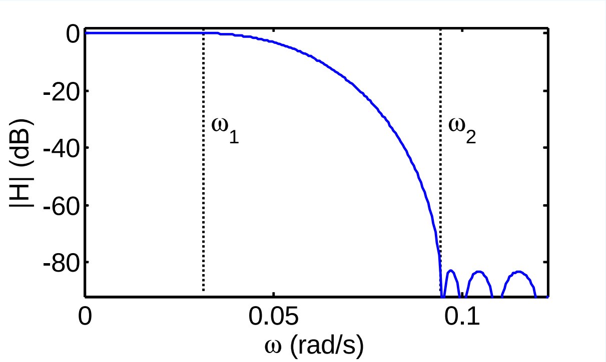
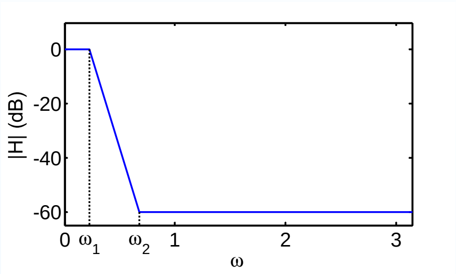




Maximum Decimation Frequency

If a filter passband occupies only a small fraction of $[0, \pi]$, we can downsample then upsample without losing information.



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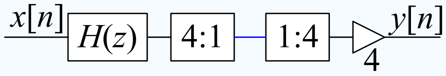

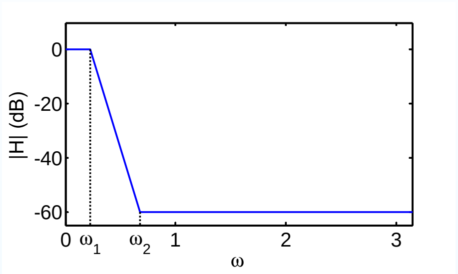

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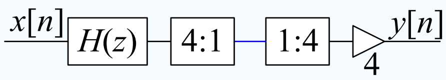

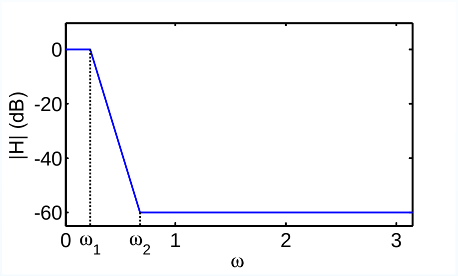

Downsample: aliased components at offsets of $\frac{2\pi}{K}$ are almost zero because of $H(z)$

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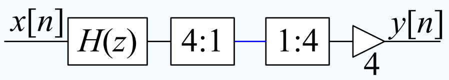

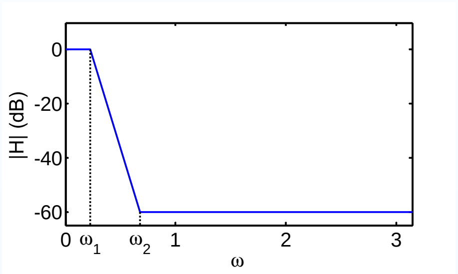

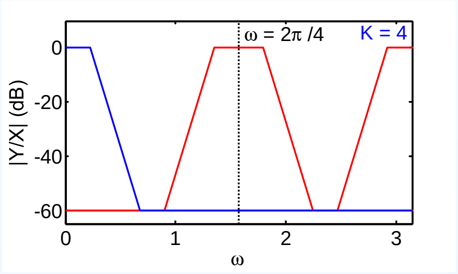

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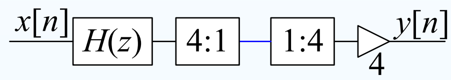

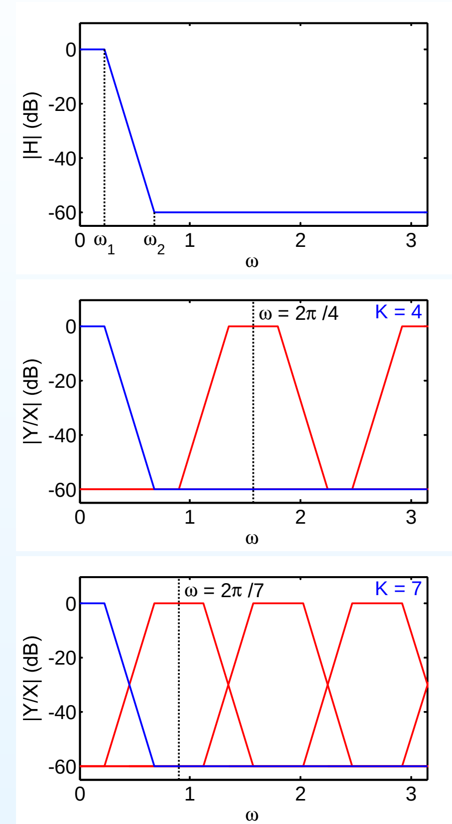

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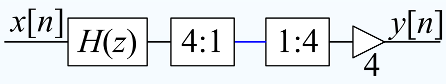

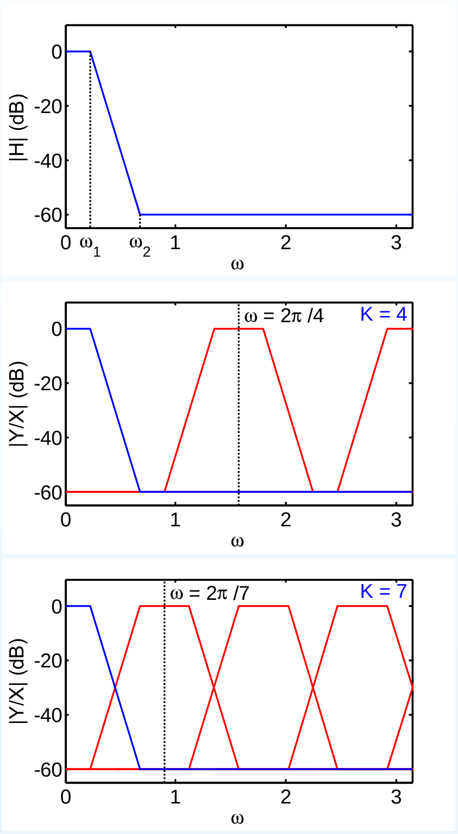

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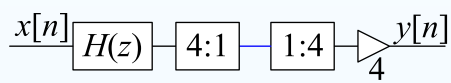

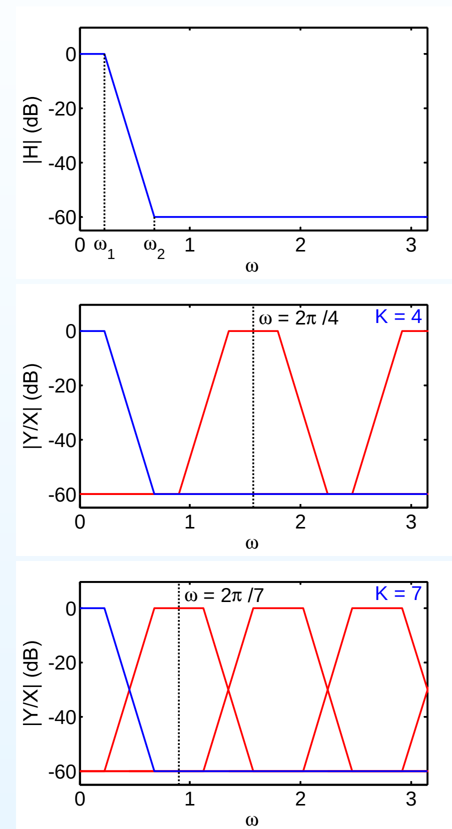

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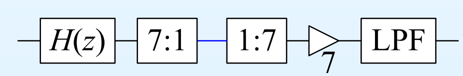
We must add a lowpass filter to remove the images:



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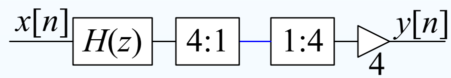

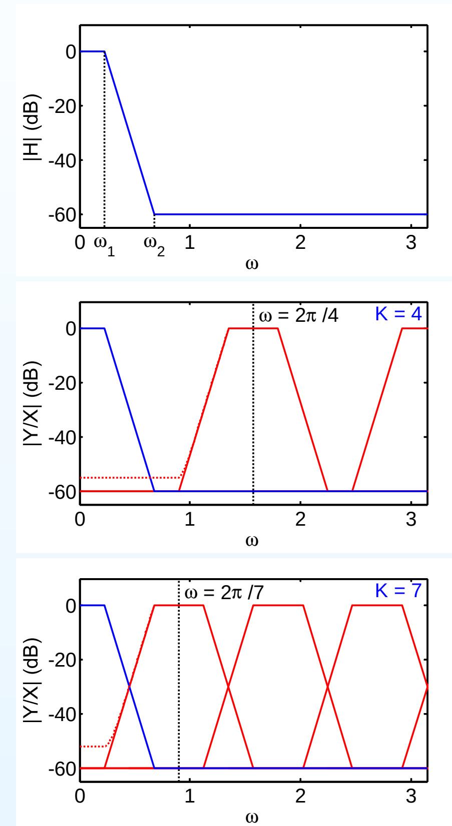

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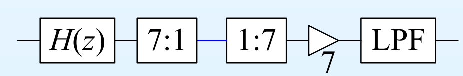

Passband noise = noise floor at output of $H(z)$ plus $10 \log_{10} (K - 1)$ dB.

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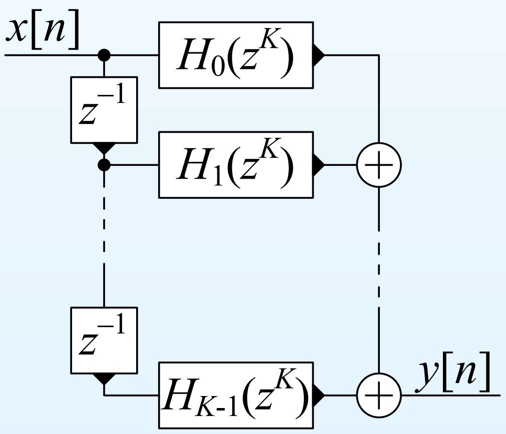

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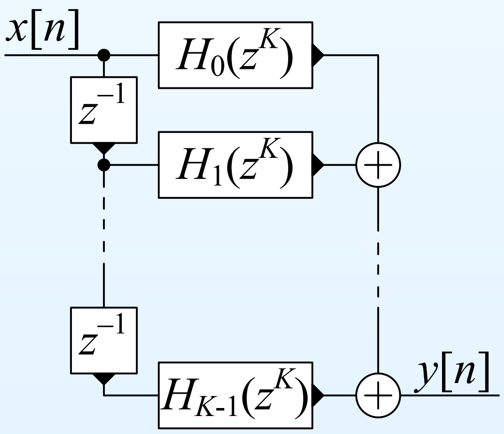

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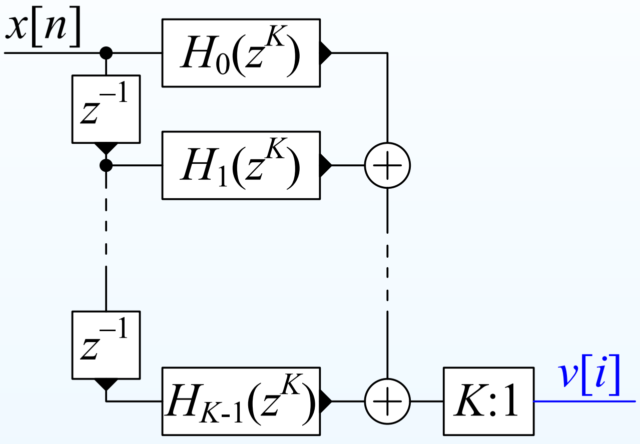
This is a polyphase implementation of the filter $H(z)$

Downsampled Polyphase Filter

$H(z)$ is low pass so we downsample its output by K without aliasing.



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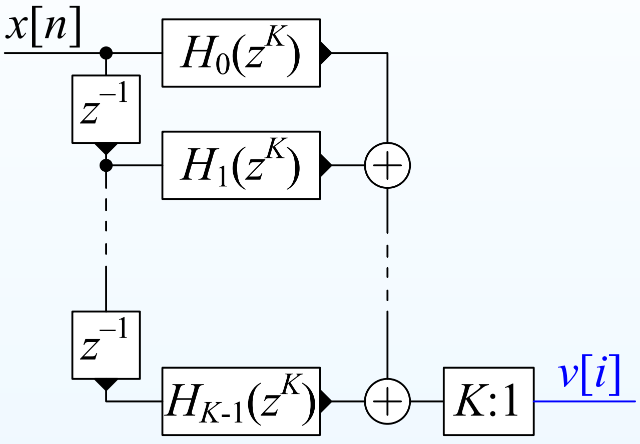
The number of multiplications per input sample is $M + 1 = 400$.



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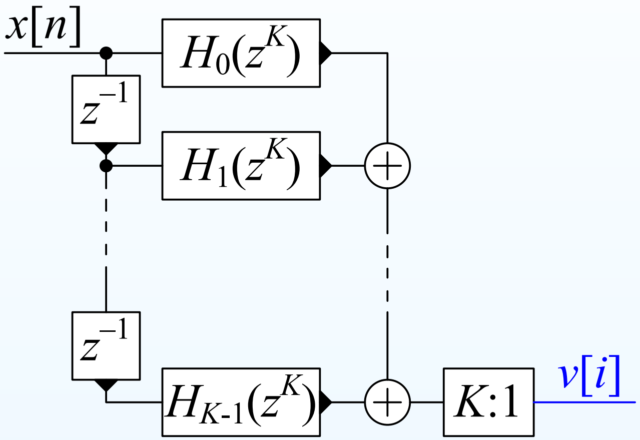

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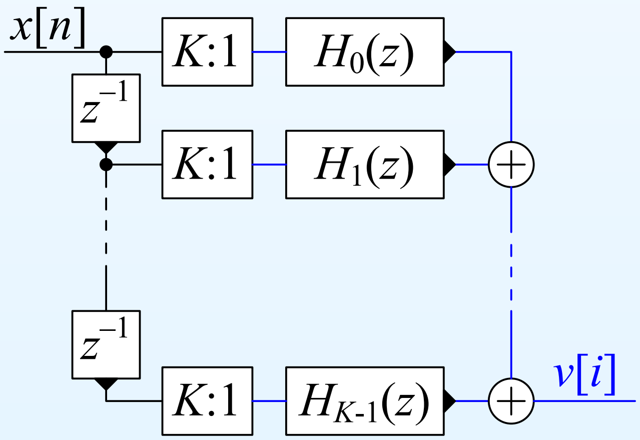
Using the Noble identities, we can move the resampling back through the adders and filters. $H_m(z^K)$ turns into $H_m(z)$ at a lower sample rate.



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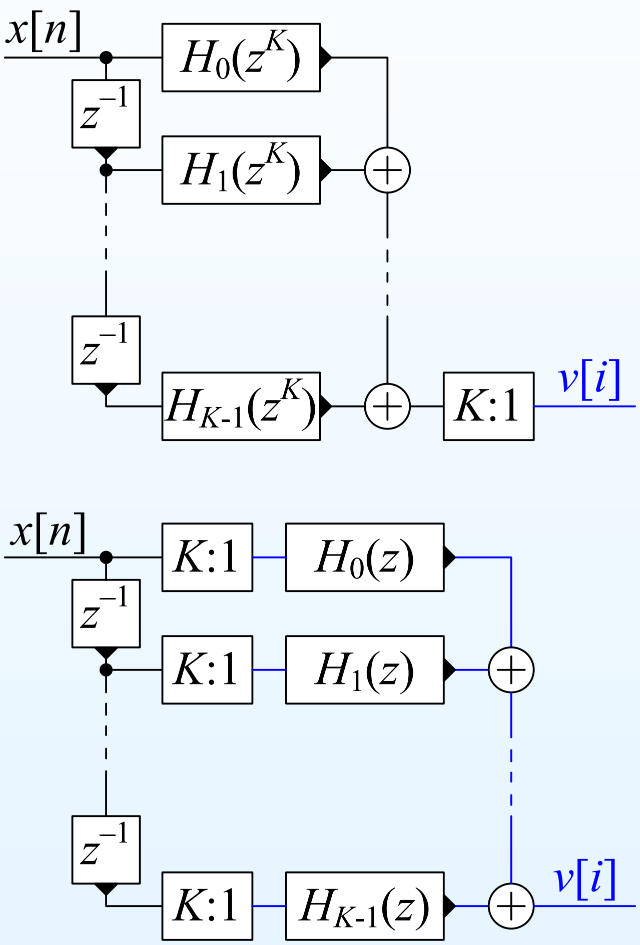

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We still perform 400 multiplications but now only once for every K input samples.

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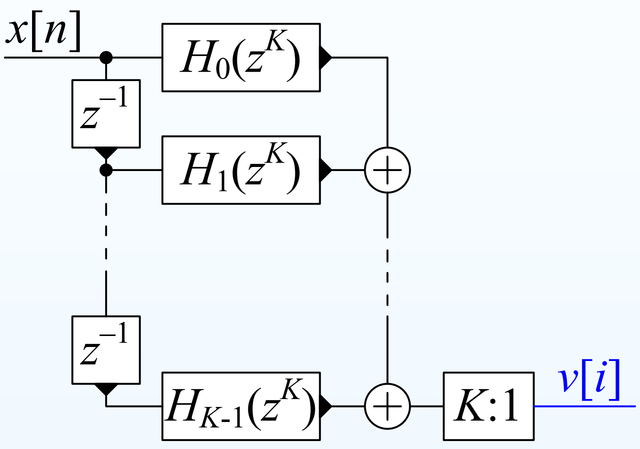
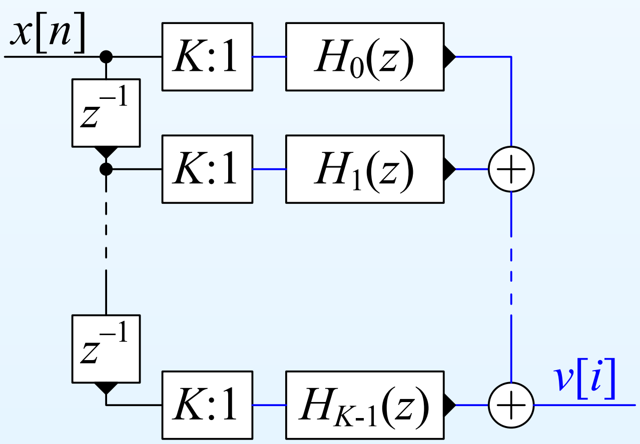
The number of multiplications per input sample is $M+1=400$.



Using the Noble identities, we can move the resampling back through the adders and filters. $H_m(z^K)$ turns into $H_m(z)$ at a lower sample rate.

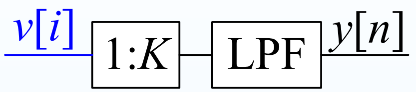
We still perform 400 multiplications but now only once for every K input samples.



Multiplications per input sample = 8 (down by a factor of 50 ☺) but $v[n]$ has the wrong sample rate (☹).

Polyphase Upsampler

To restore sample rate: upsample and then lowpass filter to remove images



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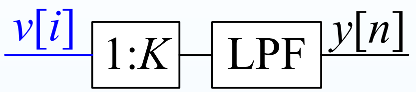

We can use the same lowpass filter, $H(z)$, in polyphase form:

$$\sum_{m=0}^{K-1} z^{-m} \sum_{r=0}^{R-1} h_m[r] z^{-Kr}$$

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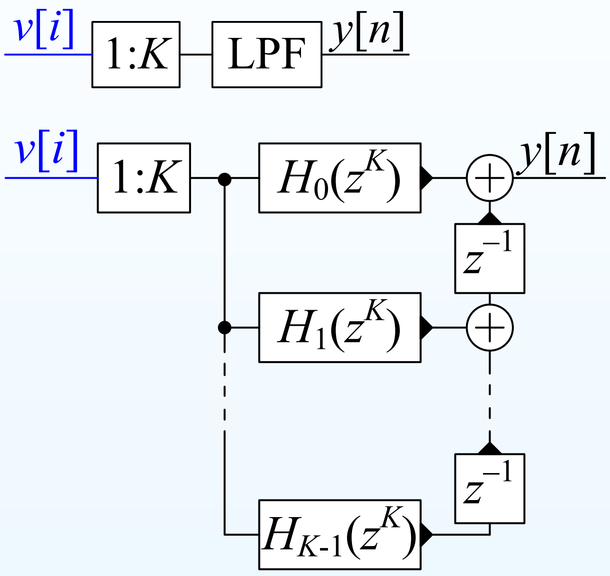

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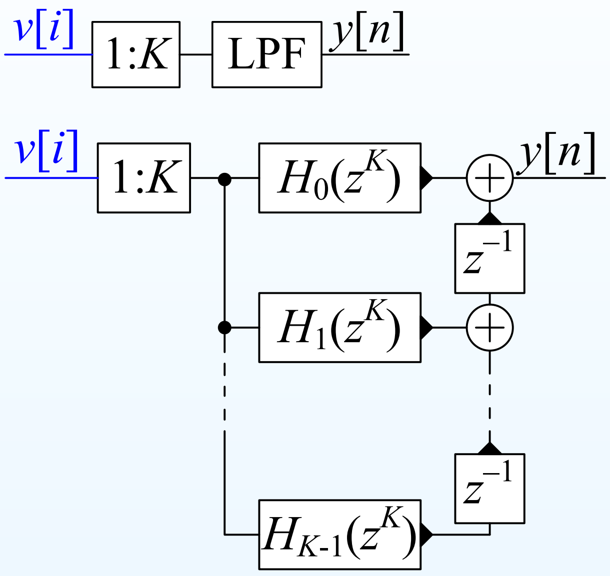
This time we put the delay z^{-m} after the filters.



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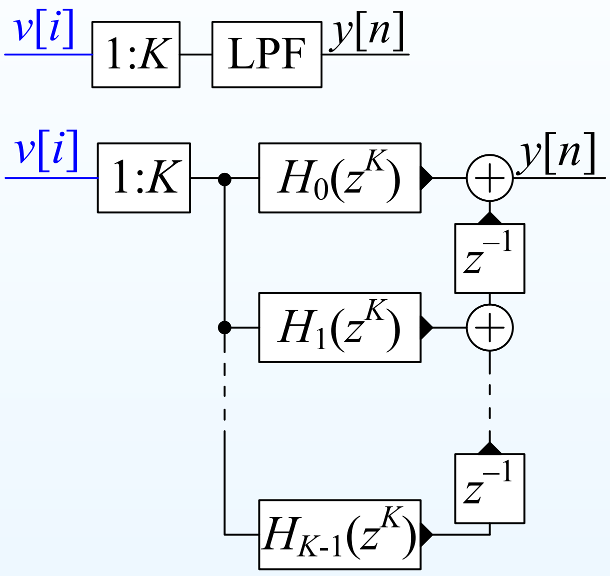
This time we put the delay z^{-m} after the filters.

Multiplications per output sample = 400



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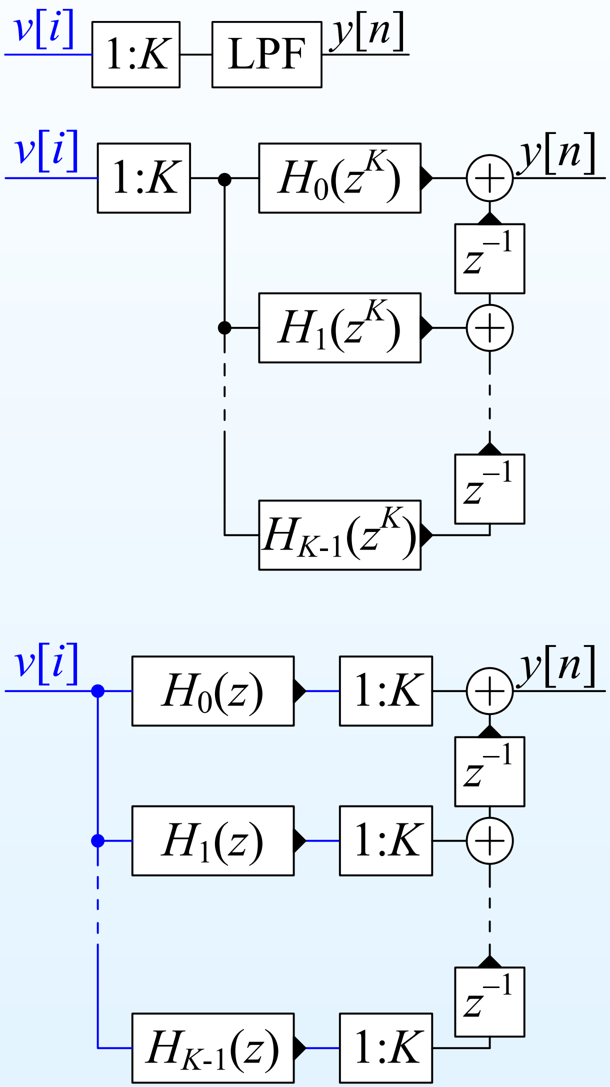
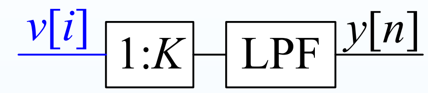
Multiplications per output sample = 400

Using the Noble identities, we can move the resampling forwards through the filters. $H_m(z^K)$ turns into $H_m(z)$ at a lower sample rate.



Polyphase Upsampler

To restore sample rate: upsample and then lowpass filter to remove images



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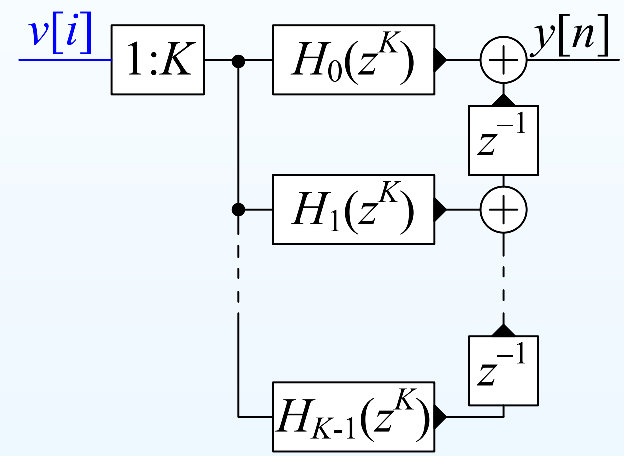

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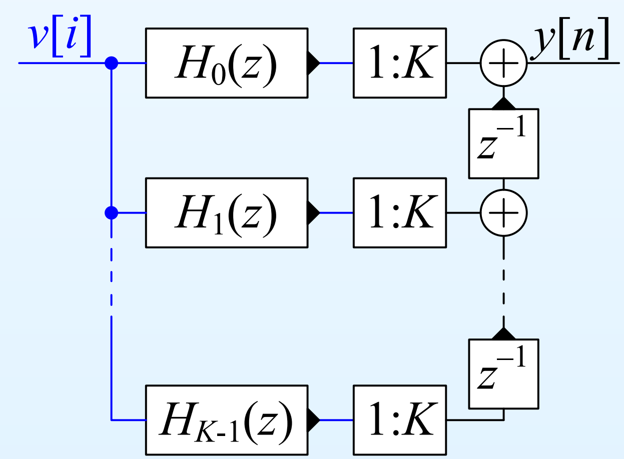
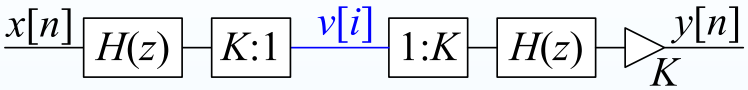


Multiplications per output sample = 8 (down by a factor of 50 ☺).

Complete Filter

The overall system implements:



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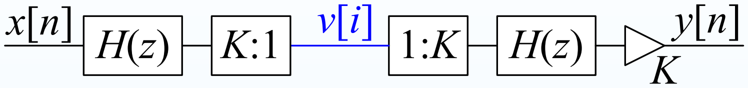
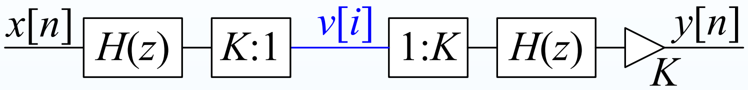


Need an extra gain of K to compensate for the downsampling energy loss.

Complete Filter

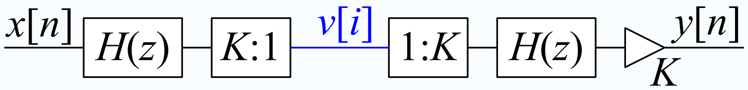
The overall system implements:



Need an extra gain of K to compensate for the downsampling energy loss.

Filtering at downsampled rate requires 16 multiplications per input sample (8 for each filter). Reduced by $\frac{K}{2}$ from the original 400.

Complete Filter

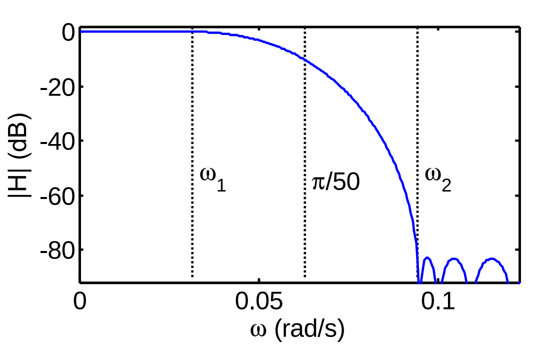


The overall system implements:

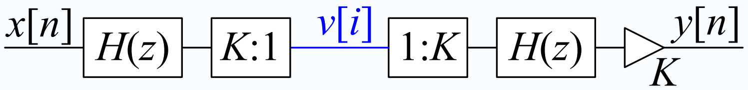
Need an extra gain of K to compensate for the downsampling energy loss.

Filtering at downsampled rate requires 16 multiplications per input sample (8 for each filter). Reduced by $\frac{K}{2}$ from the original 400.

$H(e^{j\omega})$ reaches -10 dB at the downsampler Nyquist frequency of $\frac{\pi}{K}$.



Complete Filter



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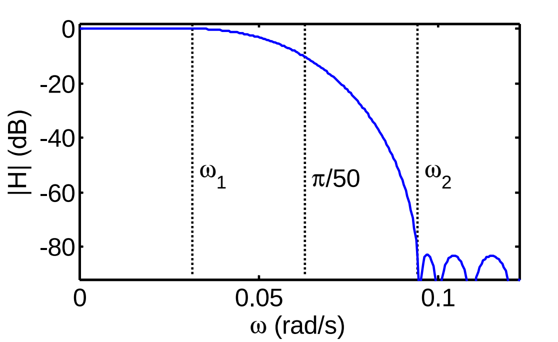
Spectral components $> \frac{\pi}{K}$ will be aliased down in frequency in $V(e^{j\omega})$.



Complete Filter

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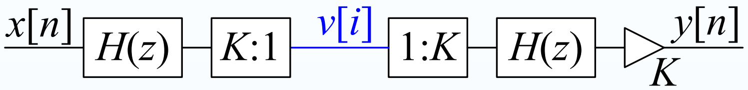

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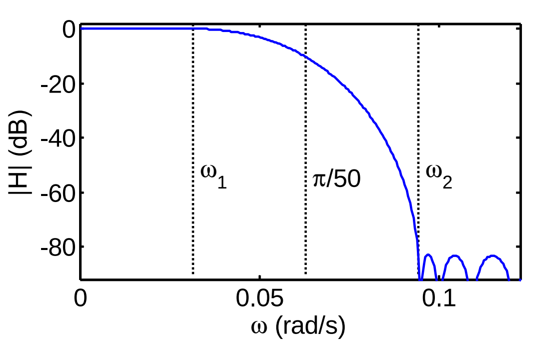
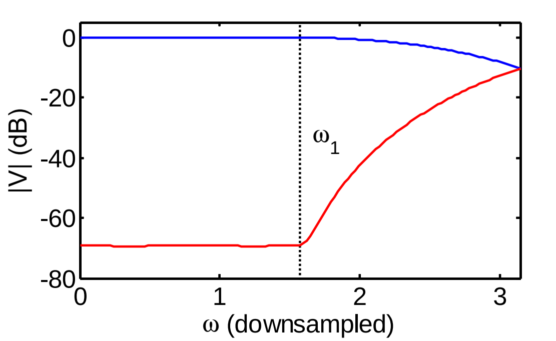
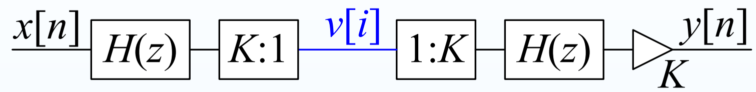


For $V(e^{j\omega})$, passband gain (blue curve) follows the same curve as $X(e^{j\omega})$.



Complete Filter



The overall system implements:
Need an extra gain of K to compensate for the downsampling energy loss.

Filtering at downsampled rate requires 16 multiplications per input sample (8 for each filter). Reduced by $\frac{K}{2}$ from the original 400.

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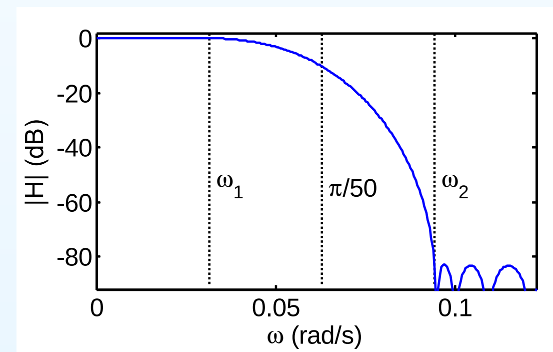
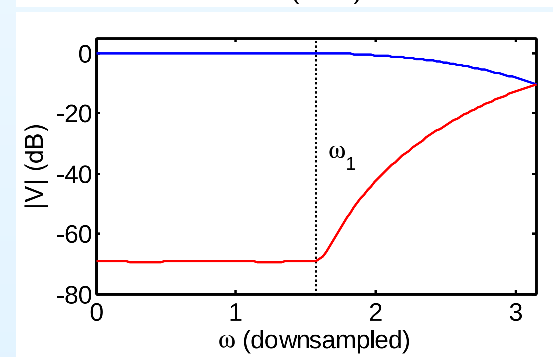
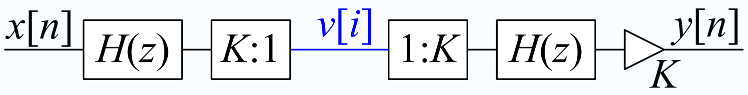


For $V(e^{j\omega})$, passband gain (blue curve) follows the same curve as $X(e^{j\omega})$.
Noise arises from K aliased spectral intervals.



Complete Filter



The overall system implements:
Need an extra gain of K to compensate for the downsampling energy loss.

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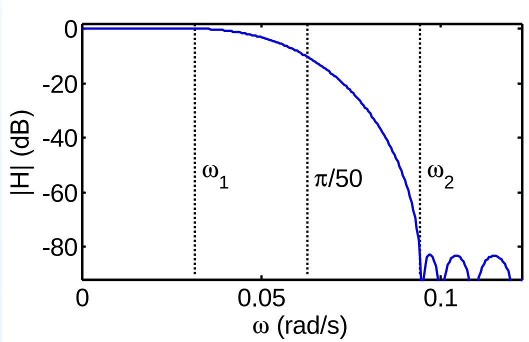
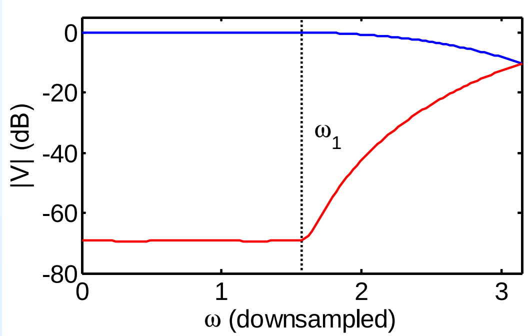
$H(e^{j\omega})$ reaches -10 dB at the downsampler Nyquist frequency of $\frac{\pi}{K}$.
Spectral components $> \frac{\pi}{K}$ will be aliased down in frequency in $V(e^{j\omega})$.



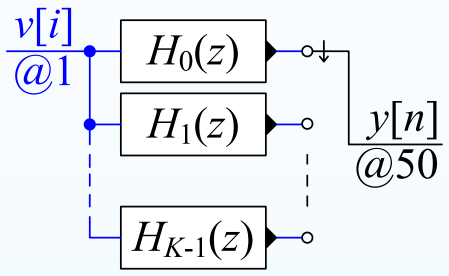
For $V(e^{j\omega})$, passband gain (blue curve) follows the same curve as $X(e^{j\omega})$.
Noise arises from K aliased spectral intervals.
Unit white noise in $X(e^{j\omega})$ gives passband noise floor at -69 dB (red curve) even though stop band ripple is below -83 dB (due to $K-1$ aliased stopband copies).



Upsampler Implementation

We can represent the upsampler compactly using a commutator. Sample $y[n]$ comes from $H_k(z)$ where $k = n \mod K$.

["$@f$" indicates the sample rate]



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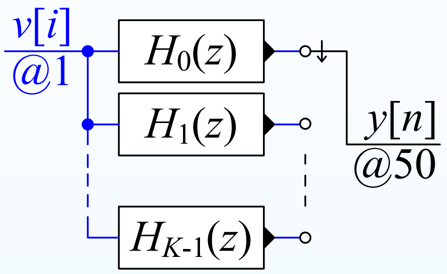
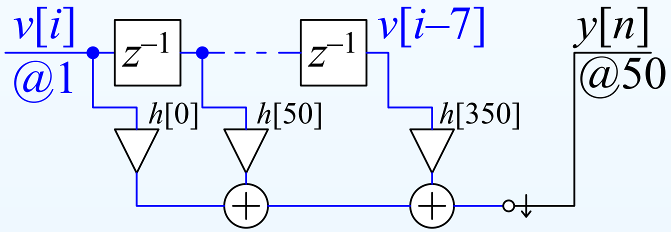


$H_0(z)$ comprises a sequence of 7 delays, 7 adders and 8 gains.



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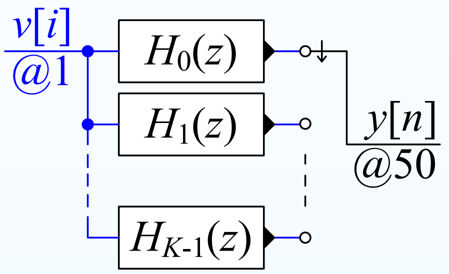
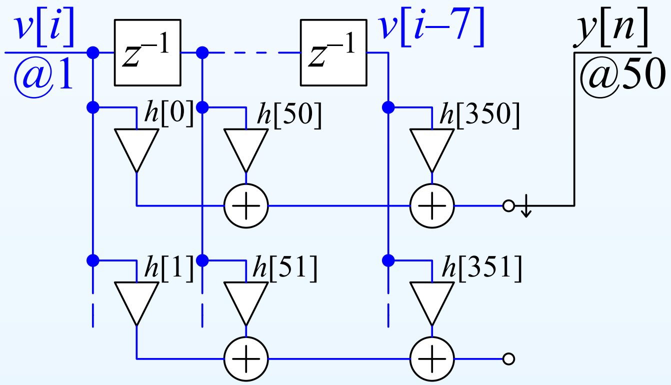


$H_0(z)$ comprises a sequence of 7 delays, 7 adders and 8 gains.

We can share the delays between all 50 filters.



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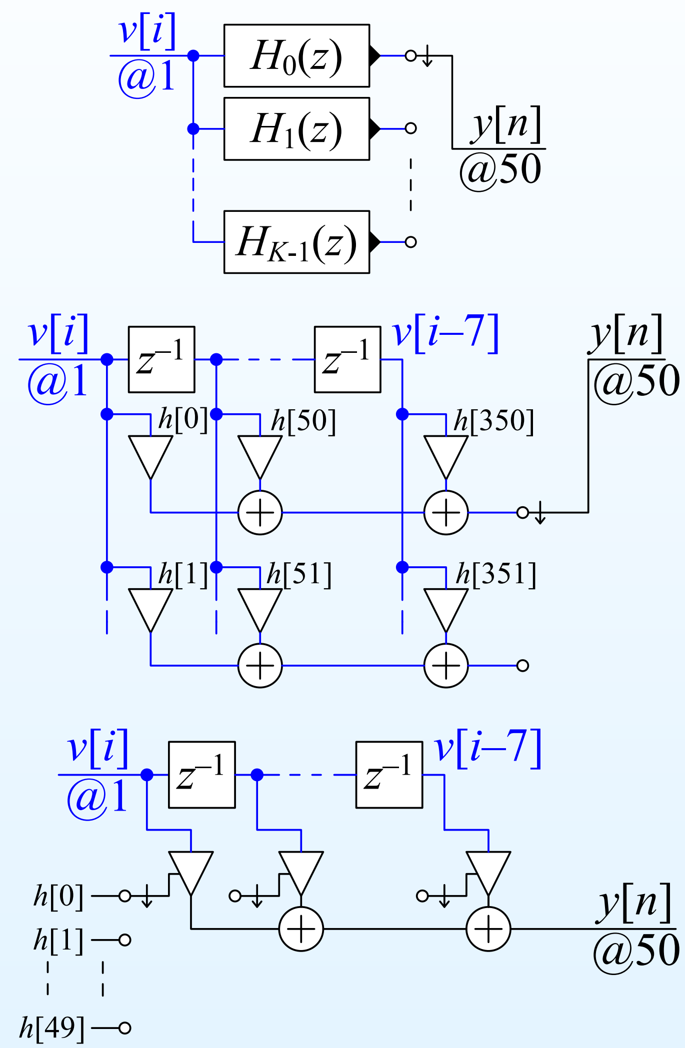
$H_0(z)$ comprises a sequence of 7 delays, 7 adders and 8 gains.

We can share the delays between all 50 filters.

We can also share the gains and adders between all 50 filters and use commutators to switch the coefficients.



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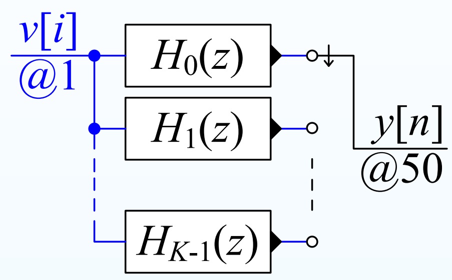

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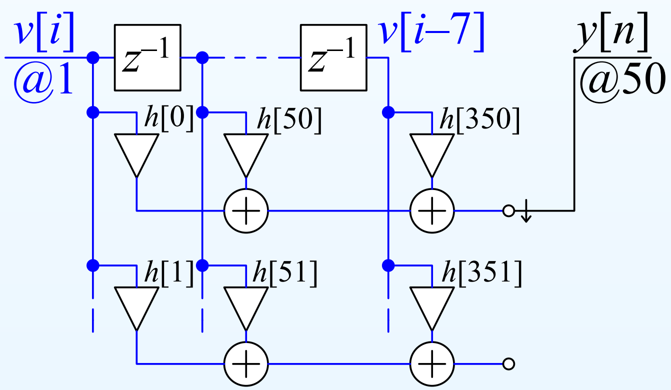
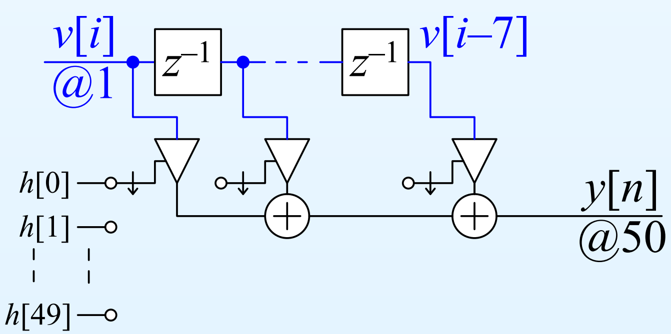


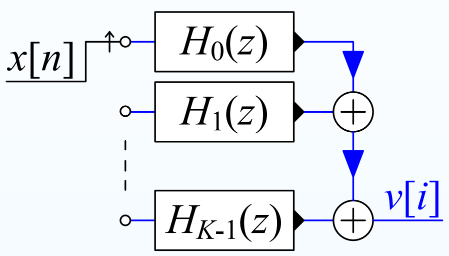
We can also share the gains and adders between all 50 filters and use commutators to switch the coefficients.



We now need 7 delays, 7 adders and 8 gains for the entire filter.

Downsampler Implementation

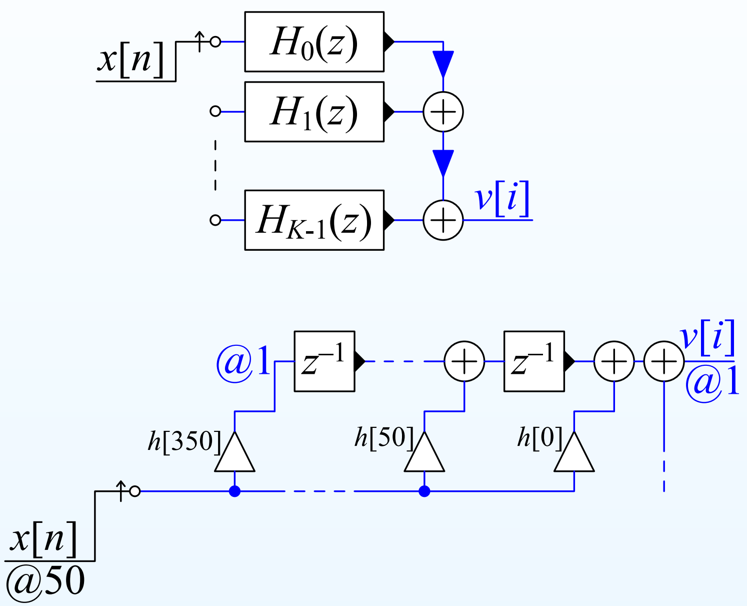
We can again use a commutator.
The outputs from all 50 filters are added together to form $v[i]$.



Downsampler Implementation

We can again use a commutator. The outputs from all 50 filters are added together to form $v[i]$.

We use the transposed form of $H_m(z)$ because this will allow us to share components.



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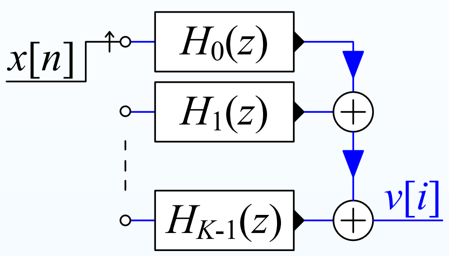

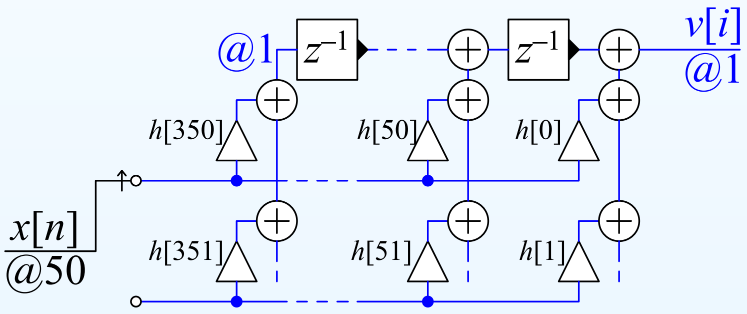

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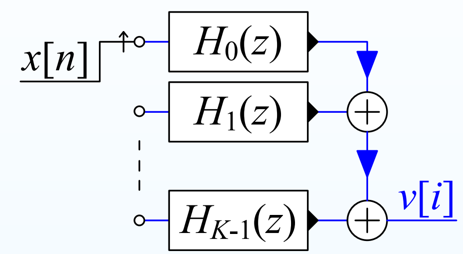

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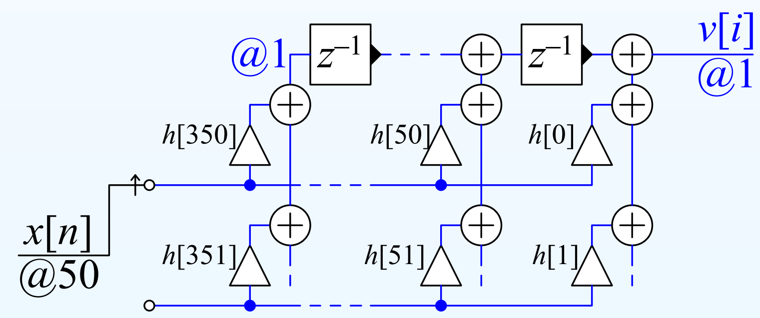
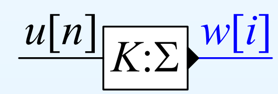


We can sum the outputs of the gain elements using an accumulator which sums blocks of K samples.



$$w[i] = \sum_{r=0}^{K-1} u[Ki - r]$$

Downsampler Implementation

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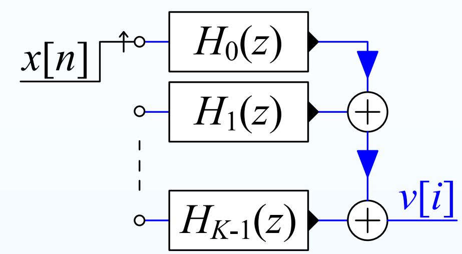

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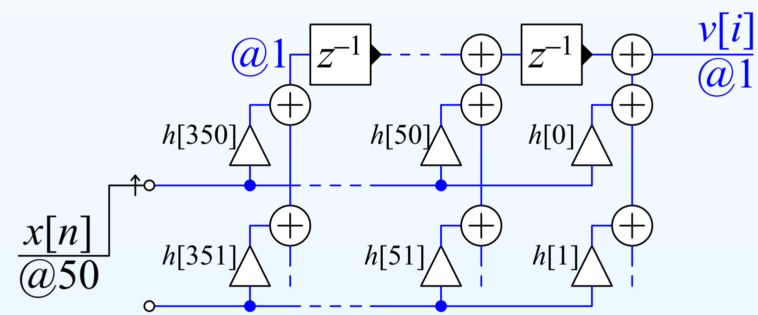

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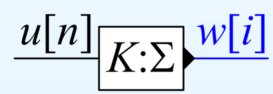
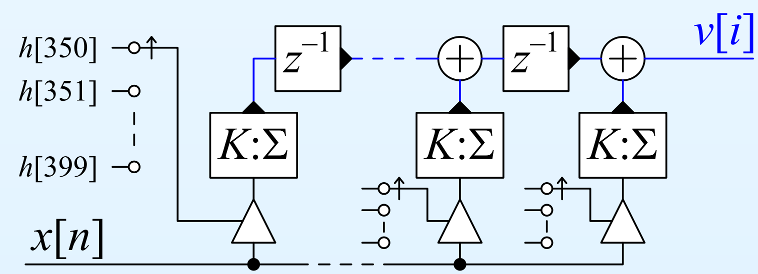


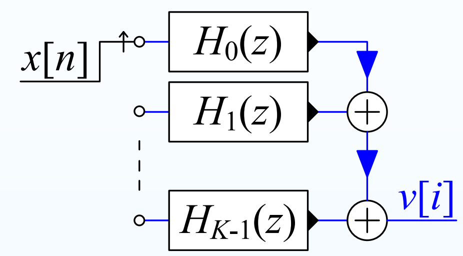
$w[i] = \sum_{r=0}^{K-1} u[Ki - r]$

Now we can share all the components and use commutators to switch the gain coefficients.



Downsampler Implementation

We can again use a commutator. The outputs from all 50 filters are added together to form $v[i]$.



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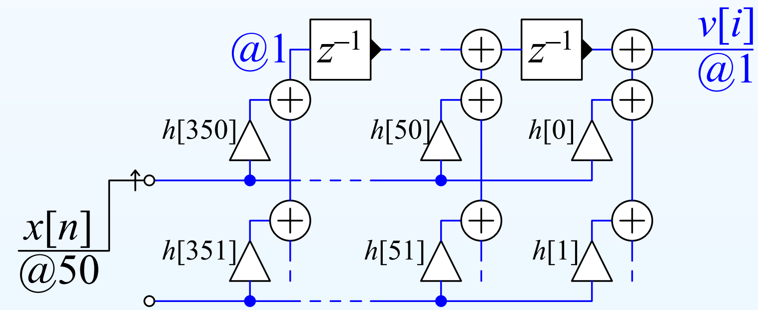

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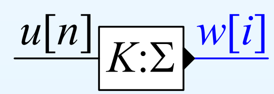

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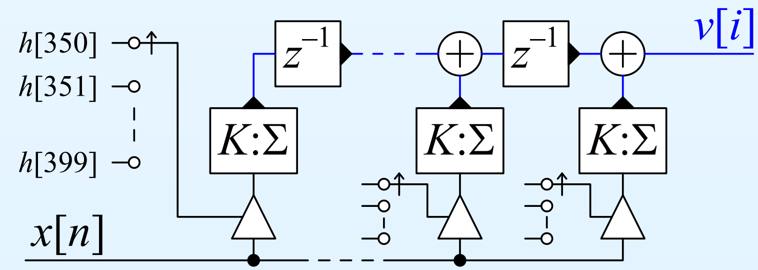

We need 7 delays, 7 adders, 8 gains and 8 accumulators in total.

Summary

- Filtering should be performed at the lowest possible sample rate
  - reduce filter computation by K
  - actual saving is only $\frac{K}{2}$ because you need a second filter
  - downsampled Nyquist frequency $\geq \max\left(\omega_{\text{passband}}\right) + \frac{\Delta\omega}{2}$

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For further details see Harris 5.