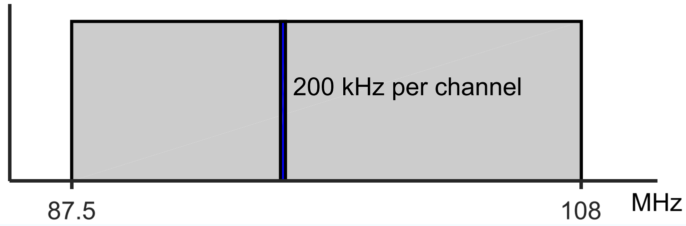
14: FM Radio Receiver

FM Radio Block Diagram

FM spectrum: 87.5 to $108\,\mathrm{MHz}$

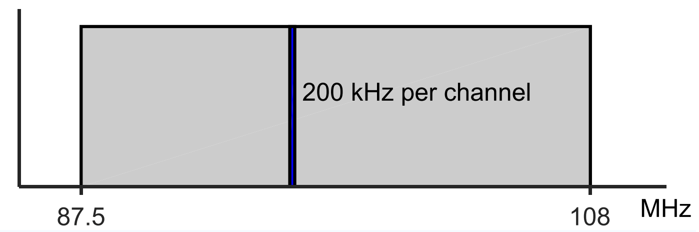


[This example is taken from Ch 13 of Harris: Multirate Signal Processing]

FM Radio Block Diagram

FM spectrum: 87.5 to $108\,\text{MHz}$

Each channel: $\pm 100\,\text{kHz}$

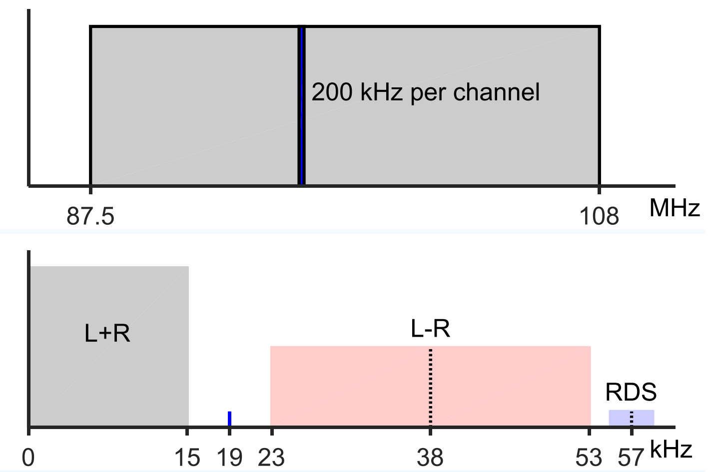


[This example is taken from Ch 13 of Harris: Multirate Signal Processing]

FM Radio Block Diagram

FM spectrum: 87.5 to $108\,\mathrm{MHz}$
Each channel: $\pm 100\,\mathrm{kHz}$

Baseband signal:
Mono (L + R): $\pm 15\,\mathrm{kHz}$



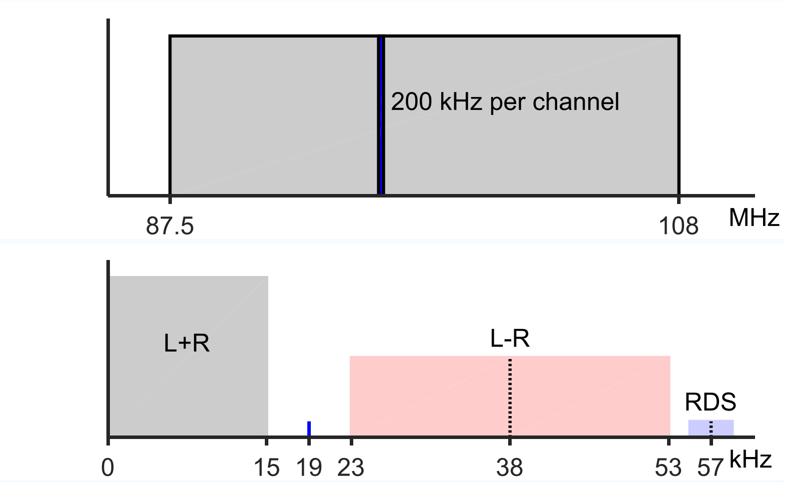
[This example is taken from Ch 13 of Harris: Multirate Signal Processing]

FM Radio Block Diagram

FM spectrum: 87.5 to $108\,\mathrm{MHz}$
Each channel: $\pm 100\,\mathrm{kHz}$

Baseband signal:
Mono (L + R): $\pm 15\,\mathrm{kHz}$

Stereo (L – R): $38 \pm 15\,\mathrm{kHz}$



[This example is taken from Ch 13 of Harris: Multirate Signal Processing]

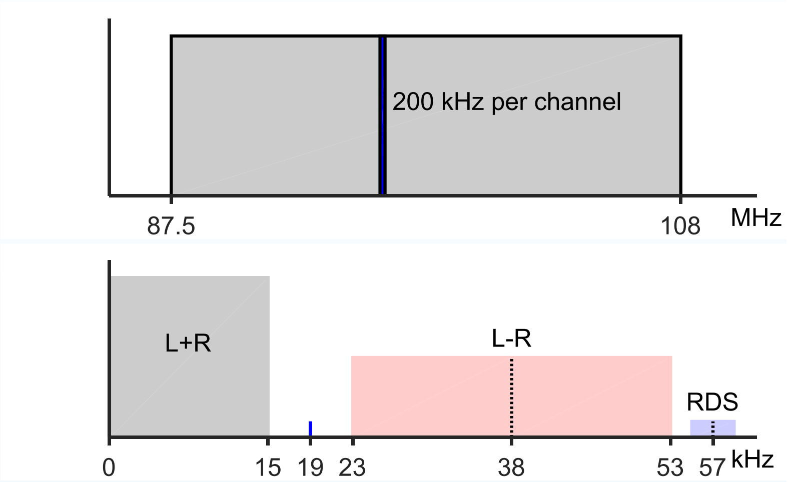
FM Radio Block Diagram

FM spectrum: 87.5 to $108\,\mathrm{MHz}$

Each channel: $\pm 100\,\mathrm{kHz}$

Baseband signal:

- Mono (L + R): $\pm 15\,\mathrm{kHz}$
- Pilot tone: $19\,\mathrm{kHz}$
- Stereo (L – R): $38 \pm 15\,\mathrm{kHz}$



[This example is taken from Ch 13 of Harris: Multirate Signal Processing]

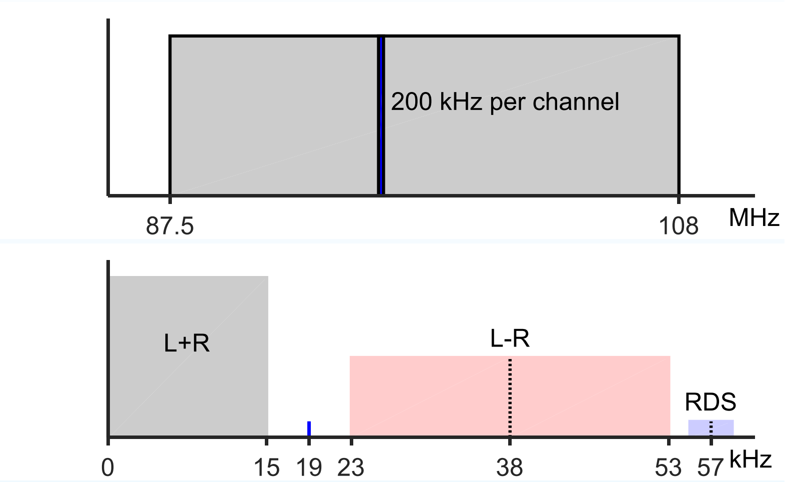
FM Radio Block Diagram

FM spectrum: 87.5 to $108\,\text{MHz}$

Each channel: $\pm 100\,\text{kHz}$

Baseband signal:

- Mono (L + R): $\pm 15\,\text{kHz}$
- Pilot tone: $19\,\text{kHz}$
- Stereo (L – R): $38 \pm 15\,\text{kHz}$
- RDS: $57 \pm 2\,\text{kHz}$



[This example is taken from Ch 13 of Harris: Multirate Signal Processing]

FM Radio Block Diagram

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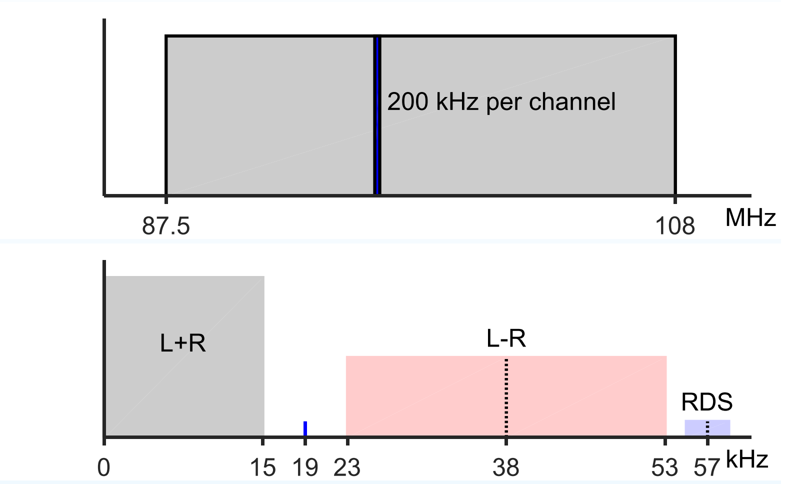

FM Modulation:

[This example is taken from Ch 13 of Harris: Multirate Signal Processing]

FM Radio Block Diagram

FM spectrum: 87.5 to $108\,\text{MHz}$
Each channel: $\pm 100\,\text{kHz}$

Baseband signal:
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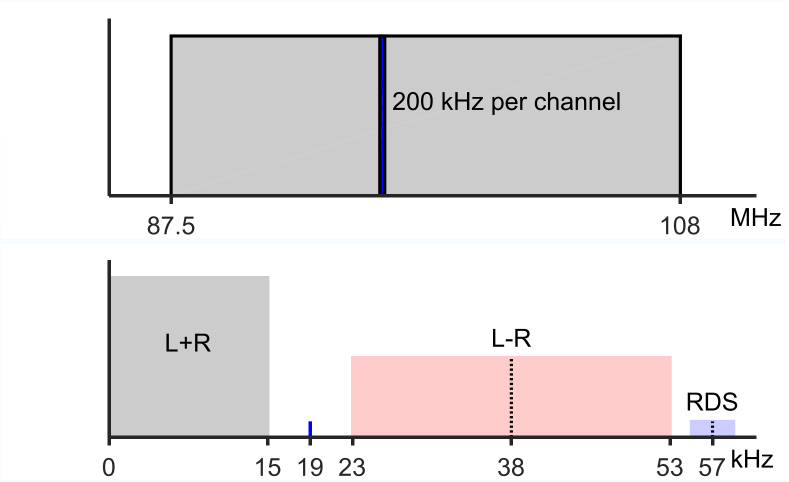
- Pilot tone: $19\,\text{kHz}$
- Stereo (L – R): $38 \pm 15\,\text{kHz}$
- RDS: $57 \pm 2\,\text{kHz}$

FM Modulation:
- Freq deviation: $\pm 75\,\text{kHz}$



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FM Radio Block Diagram

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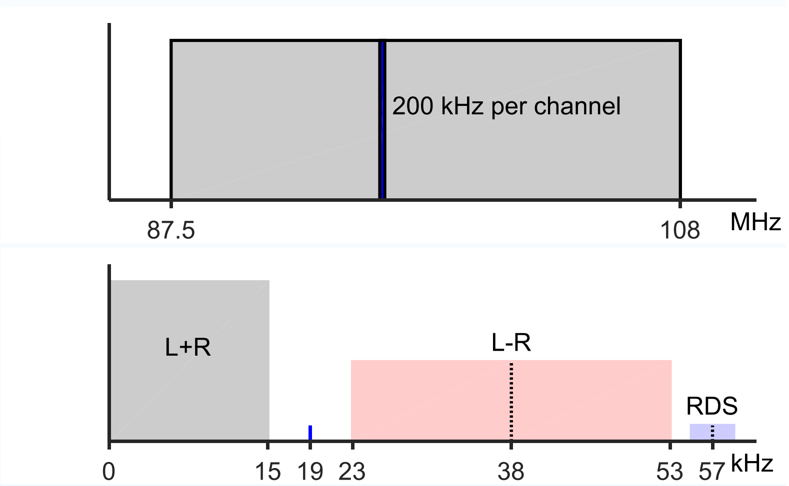
Each channel: $\pm 100\,\mathrm{kHz}$



Baseband signal:

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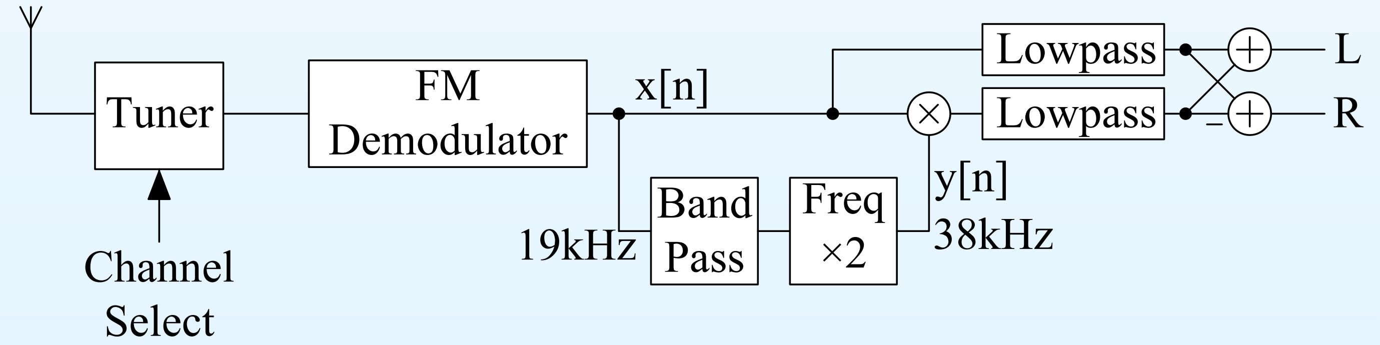
FM Modulation:

Freq deviation: $\pm 75\,\mathrm{kHz}$



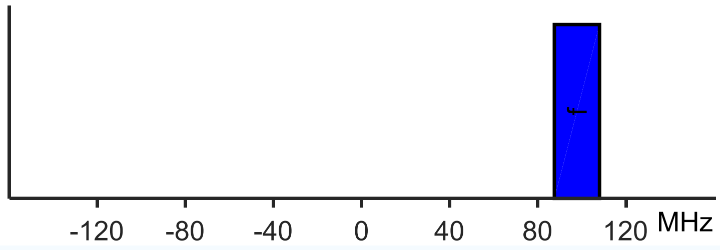
L–R signal is multiplied by $38\,\mathrm{kHz}$ to shift it to baseband

[This example is taken from Ch 13 of Harris: Multirate Signal Processing]

Aliased ADC

FM band: 87.5 to $108\,\mathrm{MHz}$

Normally sample at $f_s > 2f$

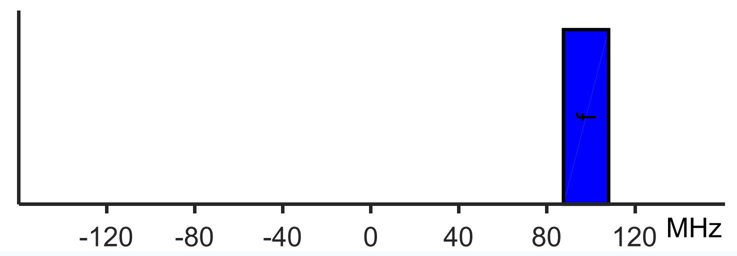


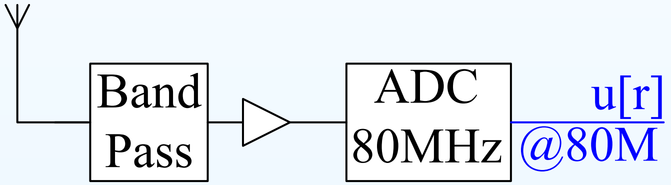
Aliased ADC

FM band: 87.5 to $108\ \mathrm{MHz}$

Normally sample at $f_s > 2f$

However:





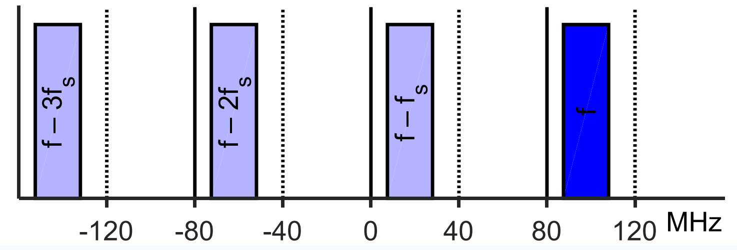
Aliased ADC

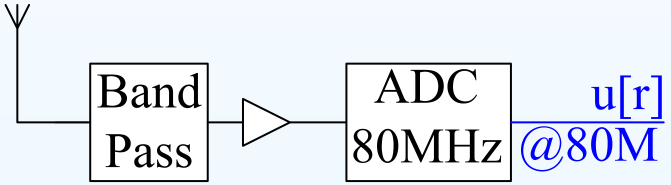
FM band: 87.5 to $108\,\mathrm{MHz}$

Normally sample at $f_s > 2f$

However:

$f_s = 80\,\mathrm{MHz}$ aliases band down to $[7.5,\ 28]\,\mathrm{MHz}$.





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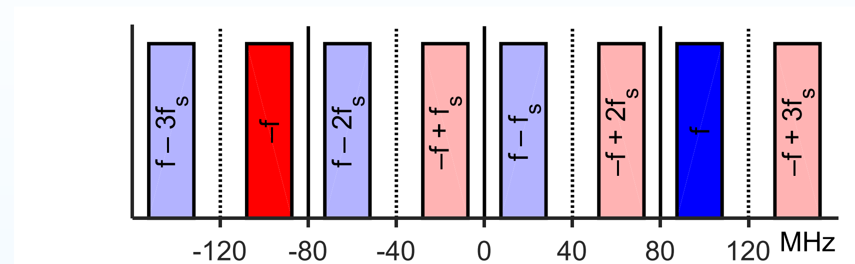

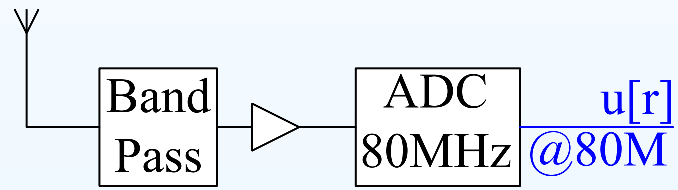

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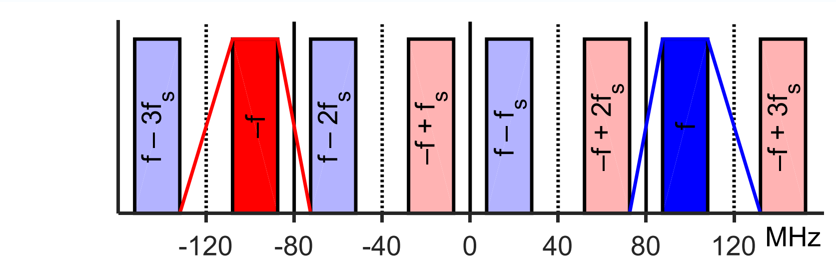
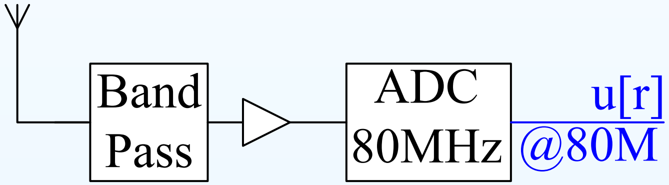
We must suppress other frequencies that alias to the range $\pm[7.5,\ 28]\,\mathrm{MHz}$.





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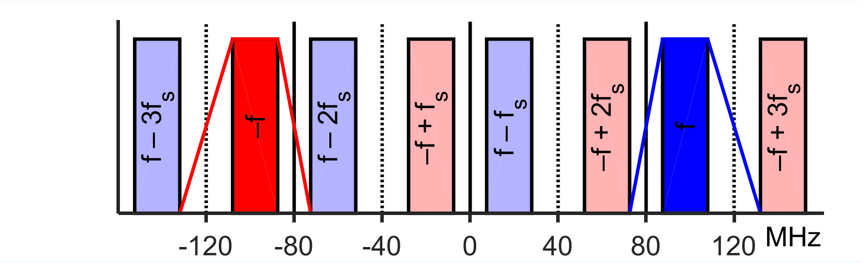

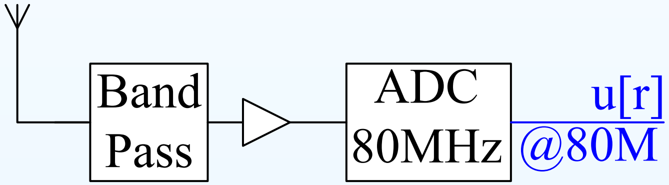

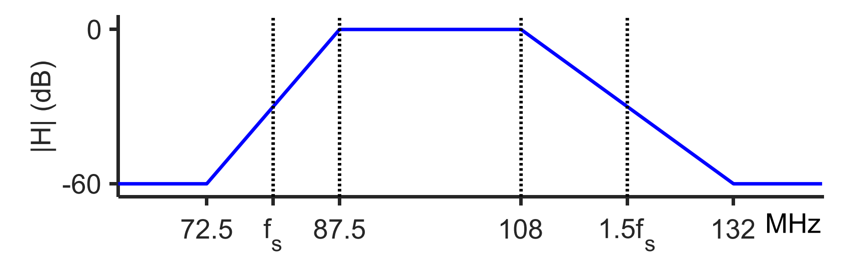

Need an analogue bandpass filter to extract the FM band.

Aliased ADC

FM band: 87.5 to $108\,\text{MHz}$
Normally sample at $f_s > 2f$

However:

$f_s = 80\,\text{MHz}$ aliases band down to $[7.5,\ 28]\,\text{MHz}$.

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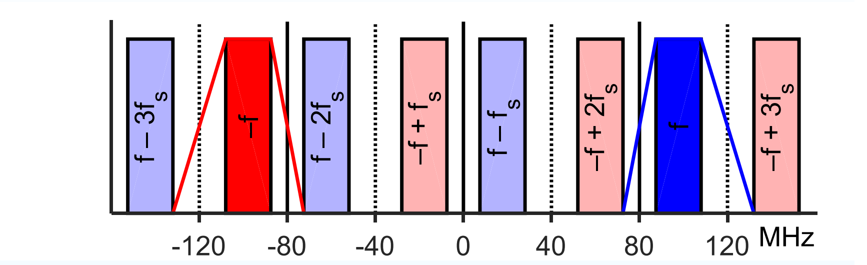

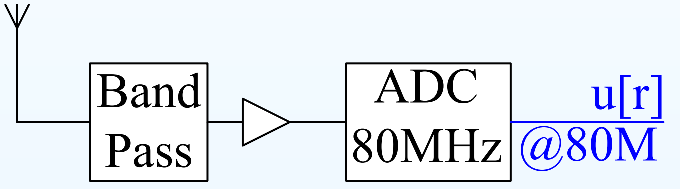

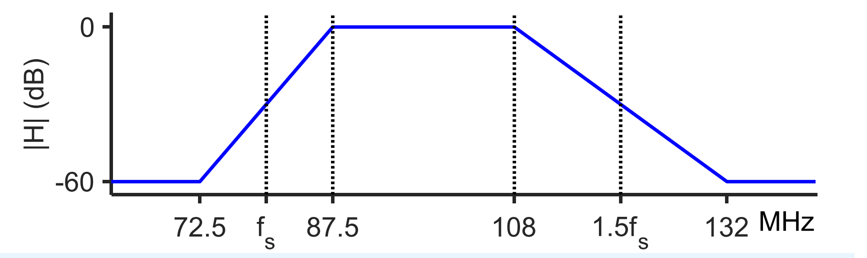

Need an analogue bandpass filter to extract the FM band. Transition band mid-points are at $f_s = 80\,\text{MHz}$ and $1.5f_s = 120\,\text{MHz}$.

Aliased ADC

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Normally sample at $f_s > 2f$

However:

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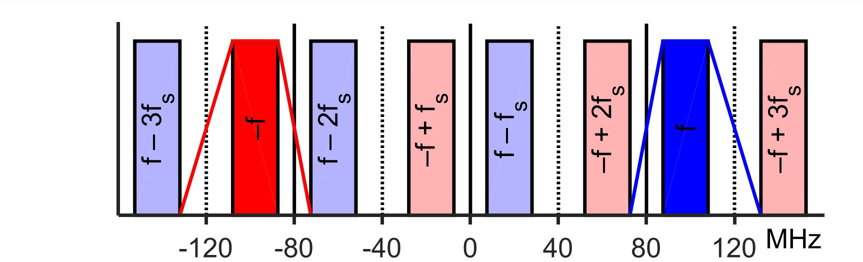

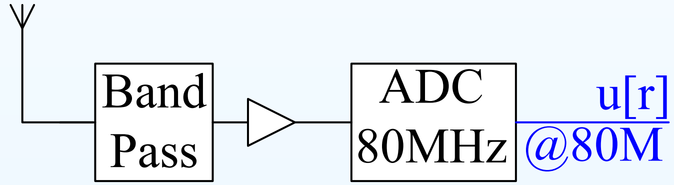

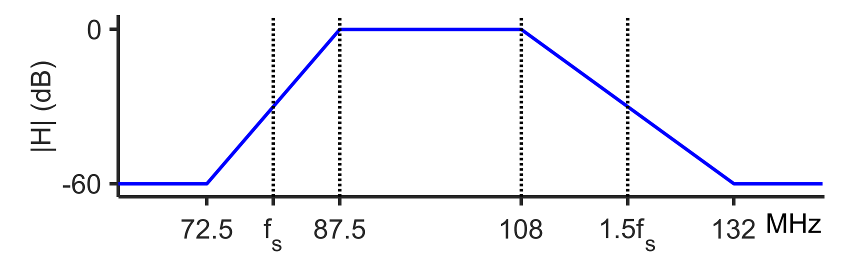

Need an analogue bandpass filter to extract the FM band. Transition band mid-points are at $f_s = 80\,\mathrm{MHz}$ and $1.5f_s = 120\,\mathrm{MHz}$.

You can use an aliased analog-digital converter (ADC) provided that the target band fits entirely between two consecutive multiples of $\frac{1}{2}f_s$.

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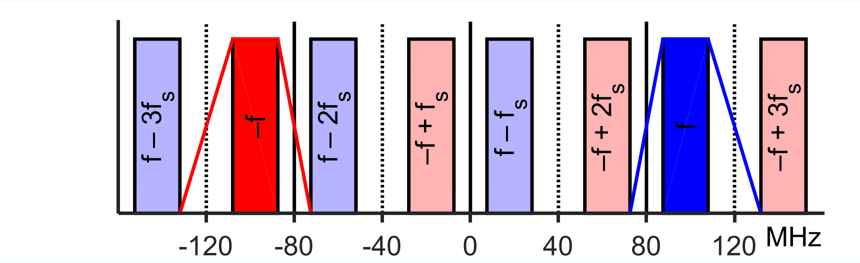
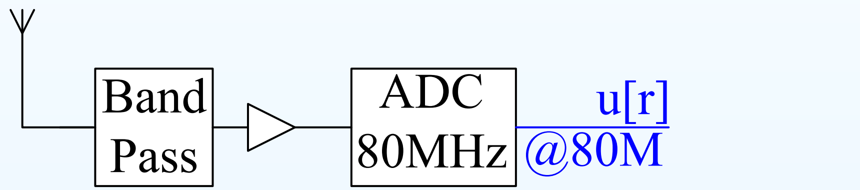
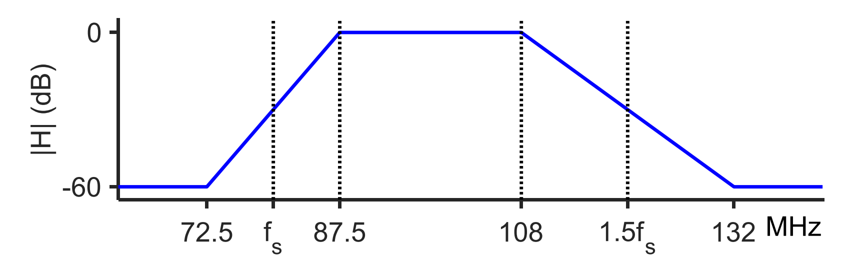




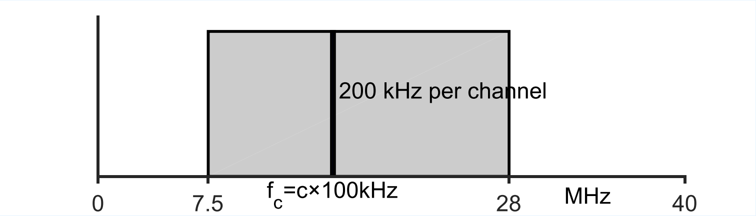


Need an analogue bandpass filter to extract the FM band. Transition band mid-points are at $f_s = 80\,\mathrm{MHz}$ and $1.5f_s = 120\,\mathrm{MHz}$.

You can use an aliased analog-digital converter (ADC) provided that the target band fits entirely between two consecutive multiples of $\frac{1}{2}f_s$.
Lower ADC sample rate ☺. Image = undistorted frequency-shifted copy.

Channel Selection

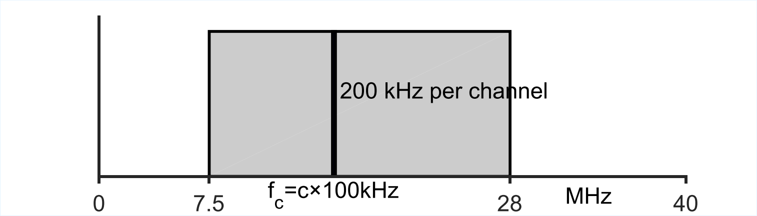
FM band shifted to 7.5 to $28\,\mathrm{MHz}$ (from 87.5 to $108\,\mathrm{MHz}$)



Channel Selection

FM band shifted to 7.5 to $28\,\mathrm{MHz}$ (from 87.5 to $108\,\mathrm{MHz}$)

We need to select a single channel $200\,\mathrm{kHz}$ wide

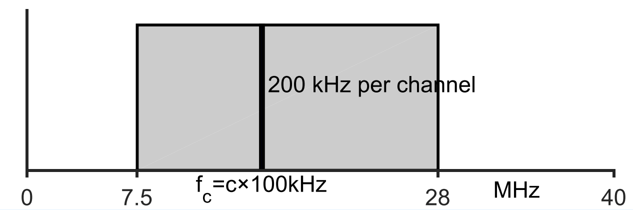


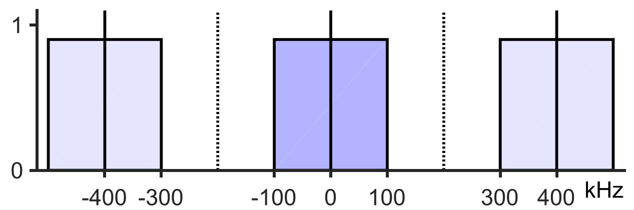
Channel Selection

FM band shifted to 7.5 to $28\,\text{MHz}$ (from 87.5 to $108\,\text{MHz}$)

We need to select a single channel $200\,\text{kHz}$ wide

We shift selected channel to DC and then downsample to $f_s = 400\,\text{kHz}$.
Assume channel centre frequency is $f_c = c \times 100\,\text{kHz}$





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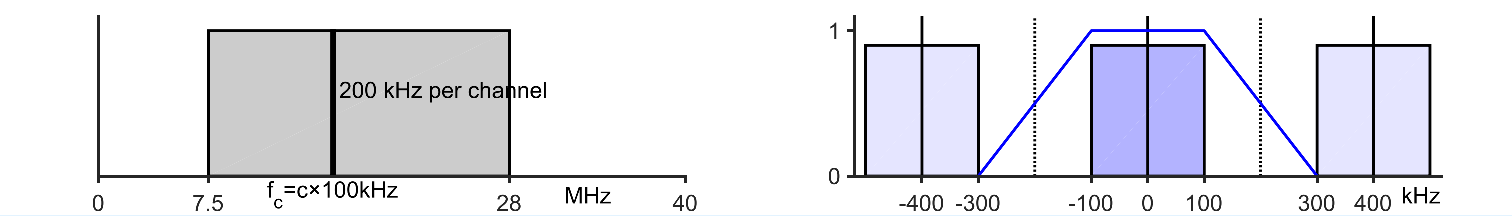
We must apply a filter before downsampling to remove unwanted images



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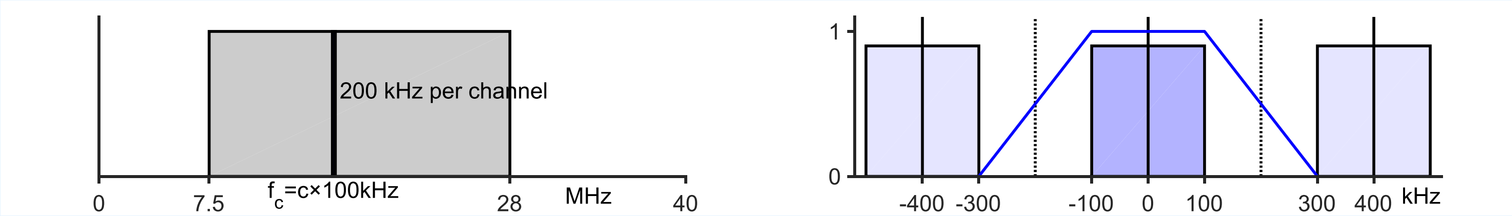
The downsampled signal is complex since positive and negative frequencies contain different information.



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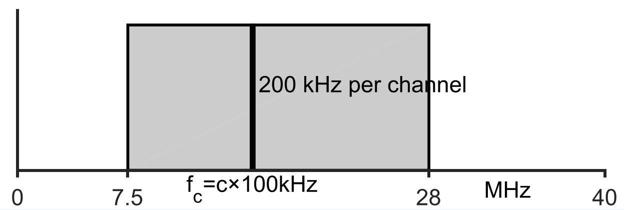
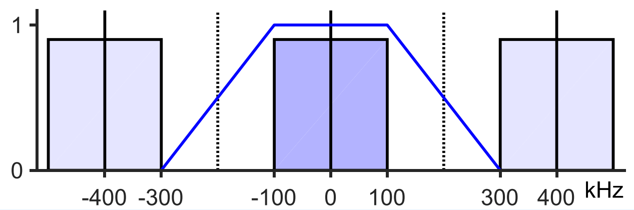
The downsampled signal is complex since positive and negative frequencies contain different information.

We will look at three methods:

1. Freq shift, then polyphase lowpass filter





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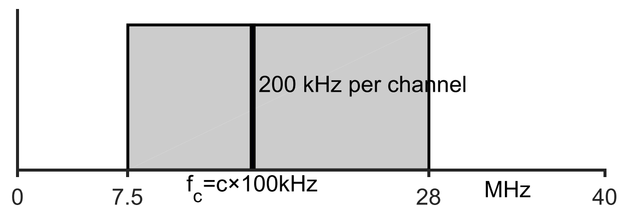
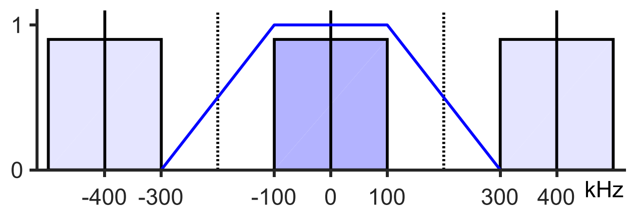
2. Polyphase bandpass complex filter





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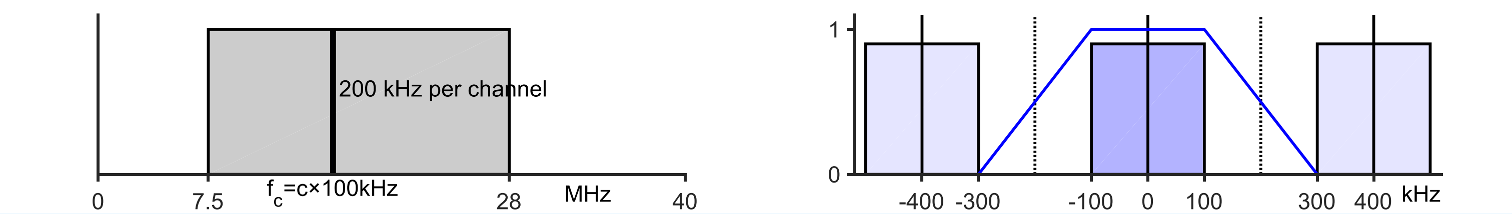
The downsampled signal is complex since positive and negative frequencies contain different information.

We will look at three methods:

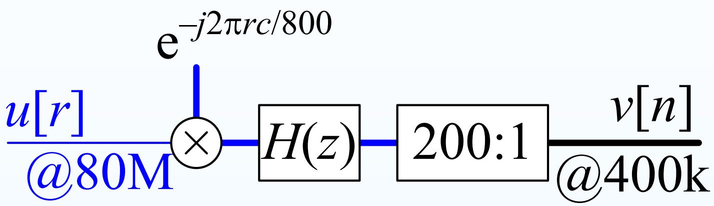
1. Freq shift, then polyphase lowpass filter
2. Polyphase bandpass complex filter
3. Polyphase bandpass real filter



Channel Selection (1)

Multiply by $e^{-j2\pi r \frac{f_c}{80\,\mathrm{MHz}}}$ to shift channel at f_c to DC.

$f_c = c \times 100\,\mathrm{k} \Rightarrow \frac{f_c}{80\,\mathrm{M}} = \frac{c}{800}$



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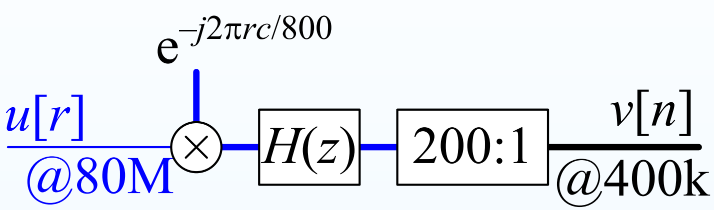
Result of multiplication is complex (thick lines on diagram)



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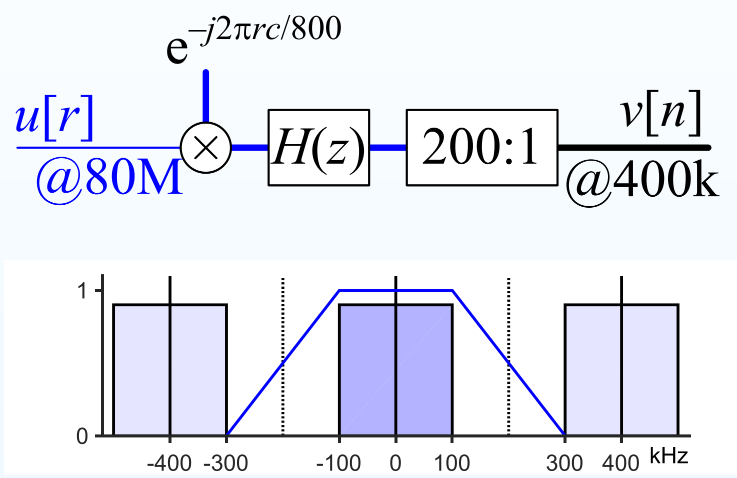
Next, lowpass filter to $\pm 100\,\mathrm{kHz}$

$\Delta\omega = 2\pi \frac{200\,\mathrm{k}}{80\,\mathrm{M}} = 0.157$



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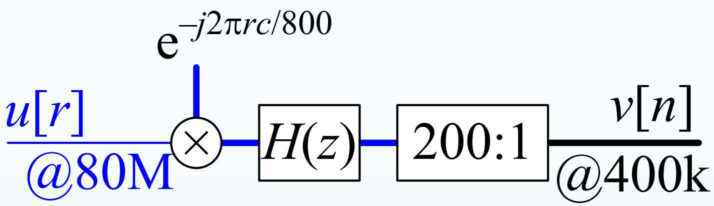
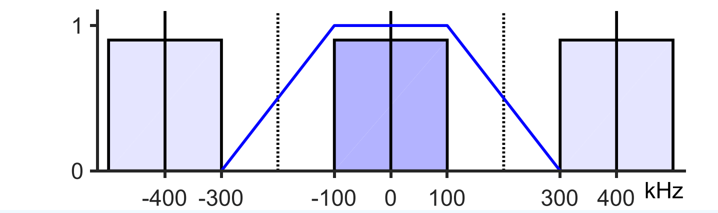
Result of multiplication is complex (thick lines on diagram)

Next, lowpass filter to $\pm 100\,\text{kHz}$

$\Delta\omega = 2\pi \frac{200\text{ k}}{80\text{ M}} = 0.157$

$\Rightarrow M = \frac{60\text{ dB}}{3.5\Delta\omega} = 1091$





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Multiply by $e^{-j2\pi r \frac{f_c}{80\,\text{MHz}}}$ to shift channel at f_c to DC.

$f_c = c \times 100\,\text{k} \Rightarrow \frac{f_c}{80\,\text{M}} = \frac{c}{800}$

Result of multiplication is complex (thick lines on diagram)

Next, lowpass filter to $\pm 100\,\text{kHz}$

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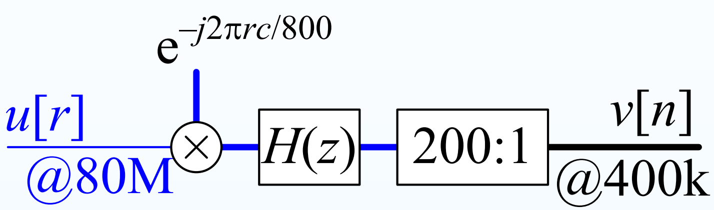

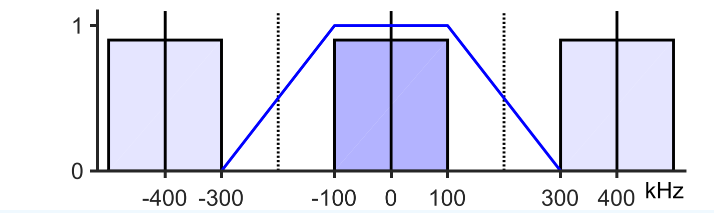

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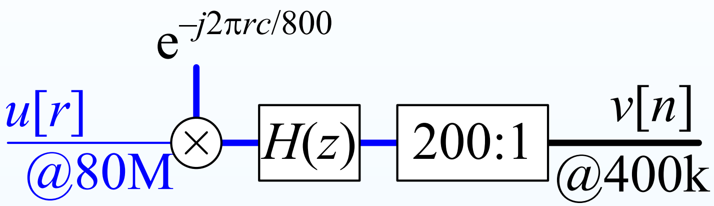

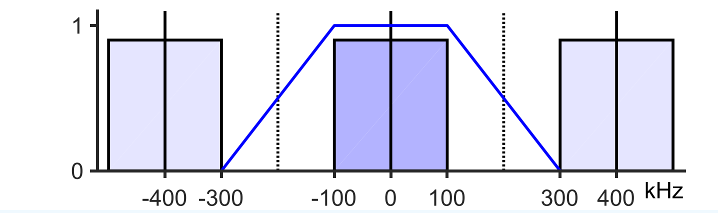

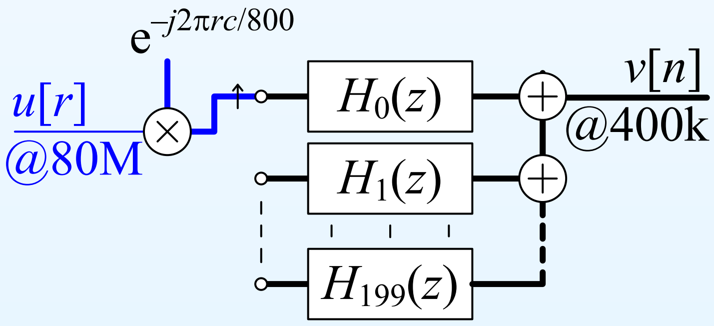

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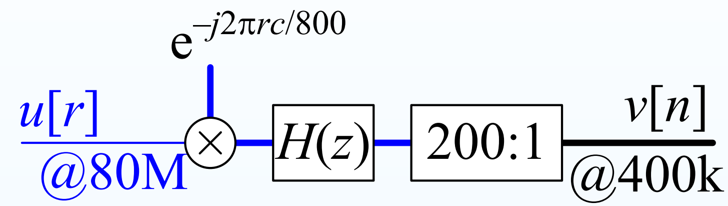

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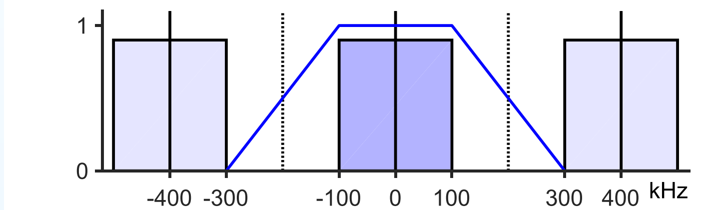

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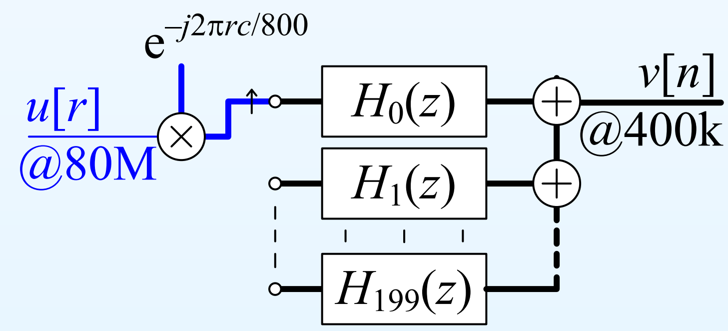

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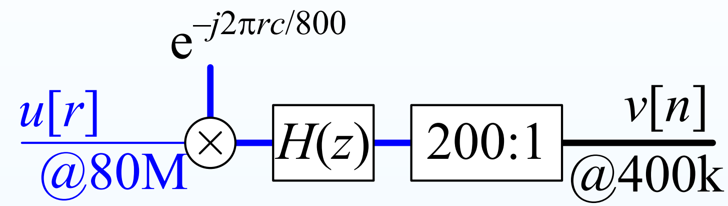

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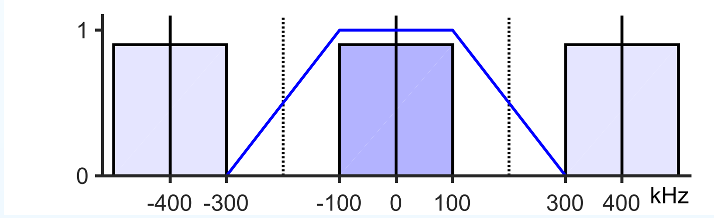

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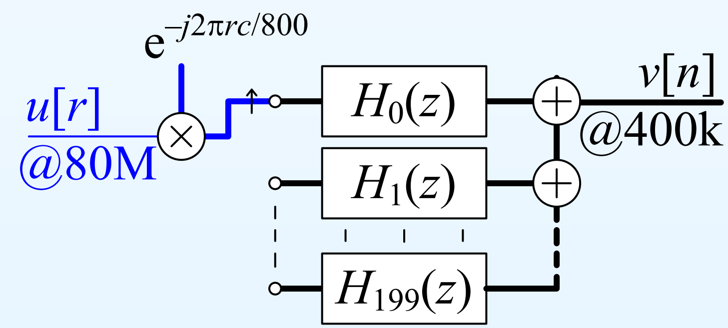

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Multiplication Load:

$2 \times 80\,\mathrm{MHz}$ (freq shift) + $12 \times 80\,\mathrm{MHz}$ ($H_p(z)$) = $14 \times 80\,\mathrm{MHz}$

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Channel centre frequency $f_c = c \times 100\,\mathrm{kHz}$ where c is an integer.

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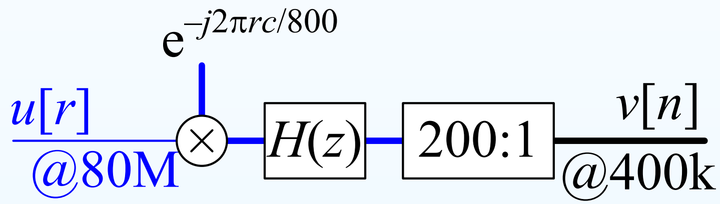

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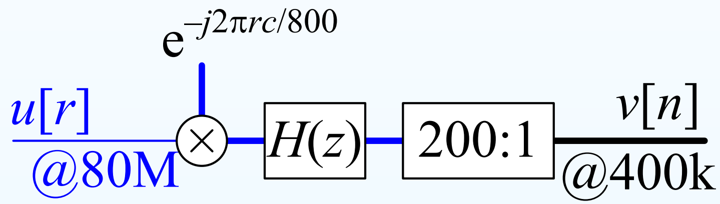

We multiply $u[r]$ by $e^{-j2\pi r \frac{c}{800}}$, convolve with $h[m]$ and then downsample:

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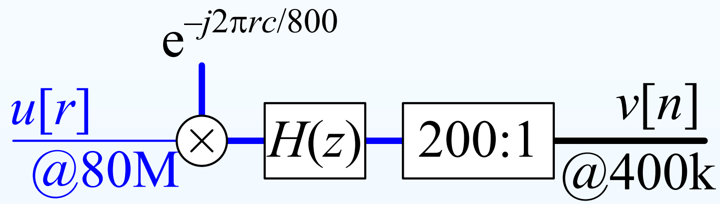

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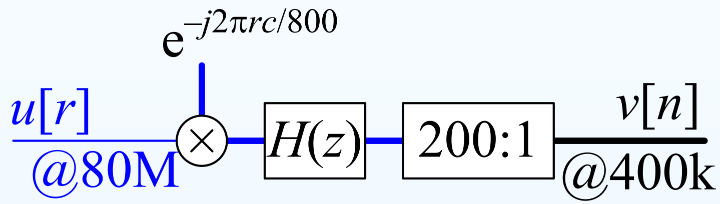

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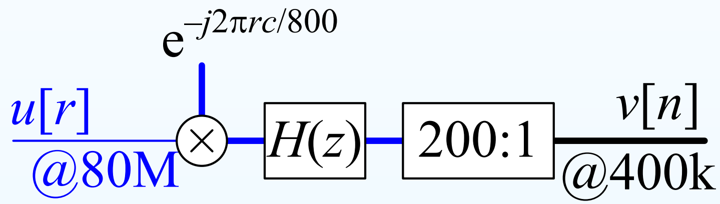

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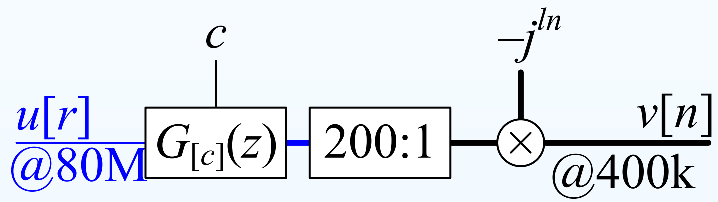

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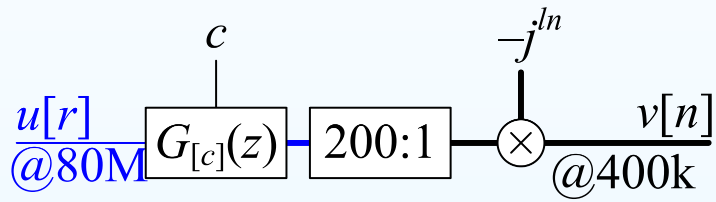

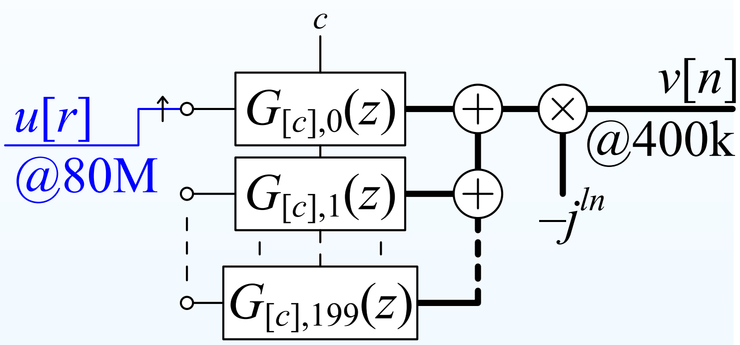

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Multiplication Load for polyphase implementation:

$G_{[c],p}(z)$ has complex coefficients $\times$ real input $\Rightarrow$ 2 mults per tap

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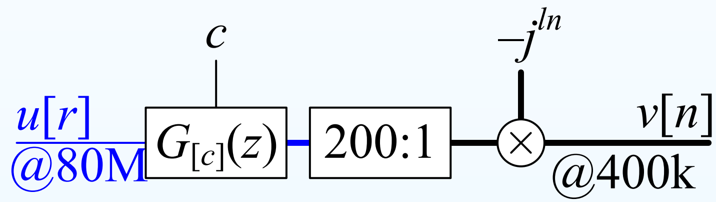

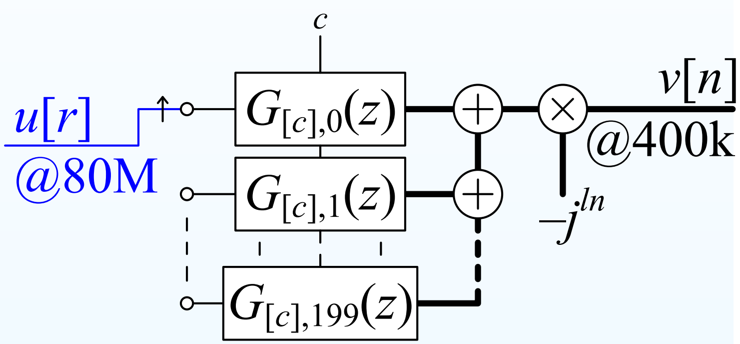

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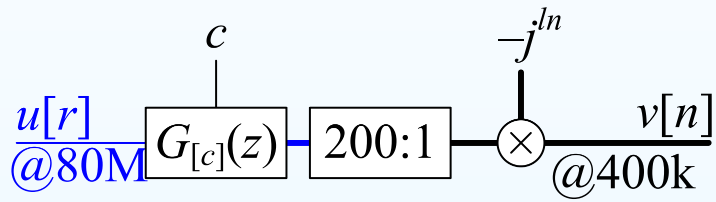

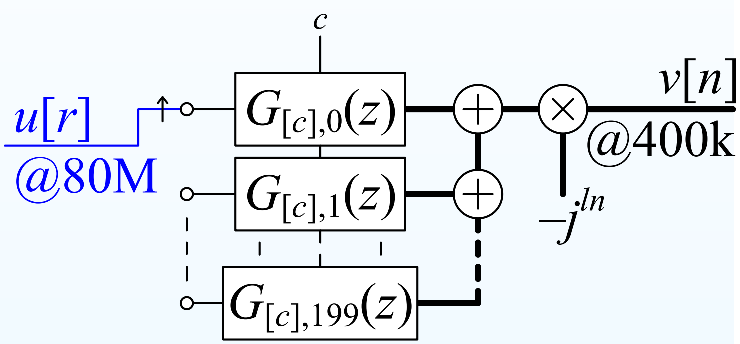

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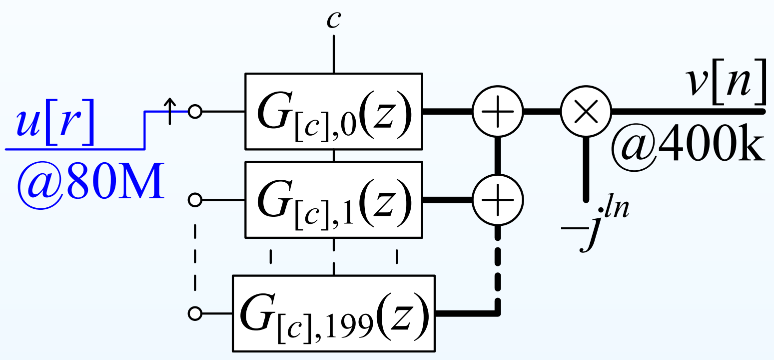
Total: $12 \times 80\,\text{MHz}$ (for $G_{[c],p}(z)$) + 0 (for $-j^{ln}$) = $12 \times 80\,\text{MHz}$

Channel Selection (3)

Channel frequency $f_c = c \times 100\,\text{kHz}$ where $c = 4k + l$ is an integer



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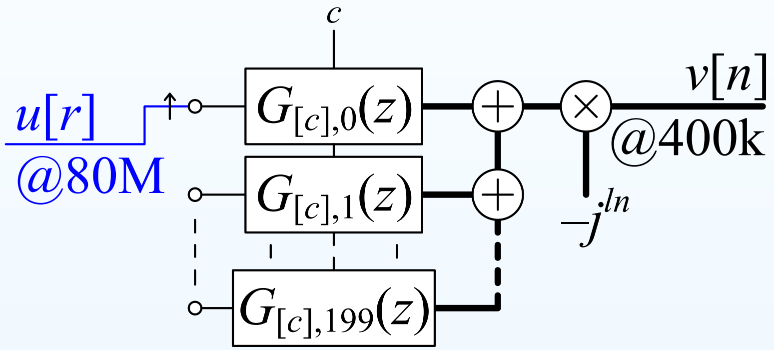

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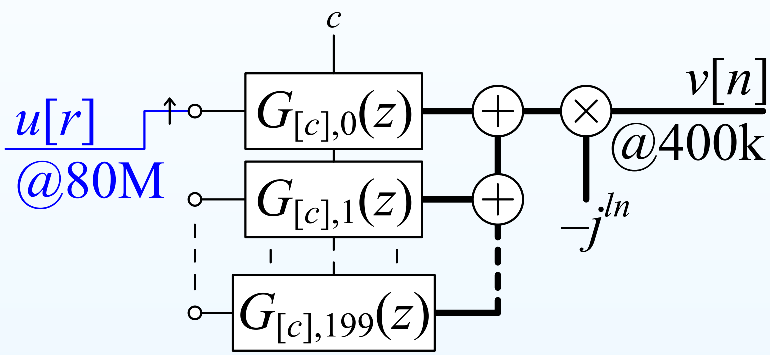

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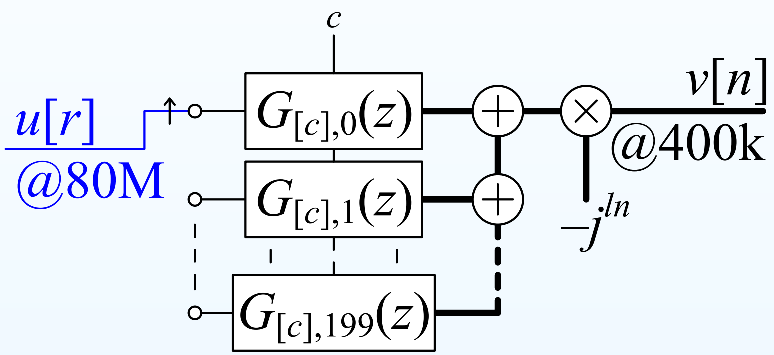

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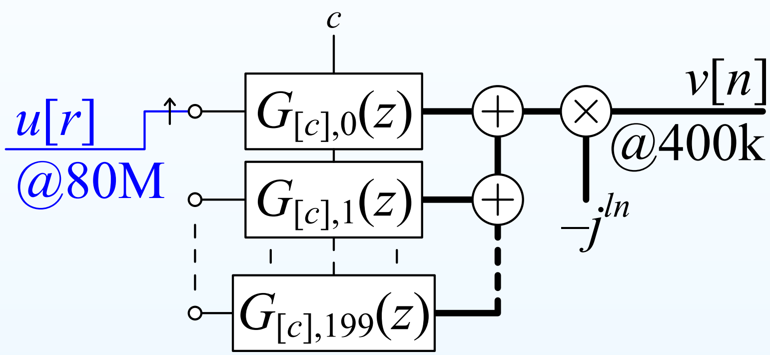

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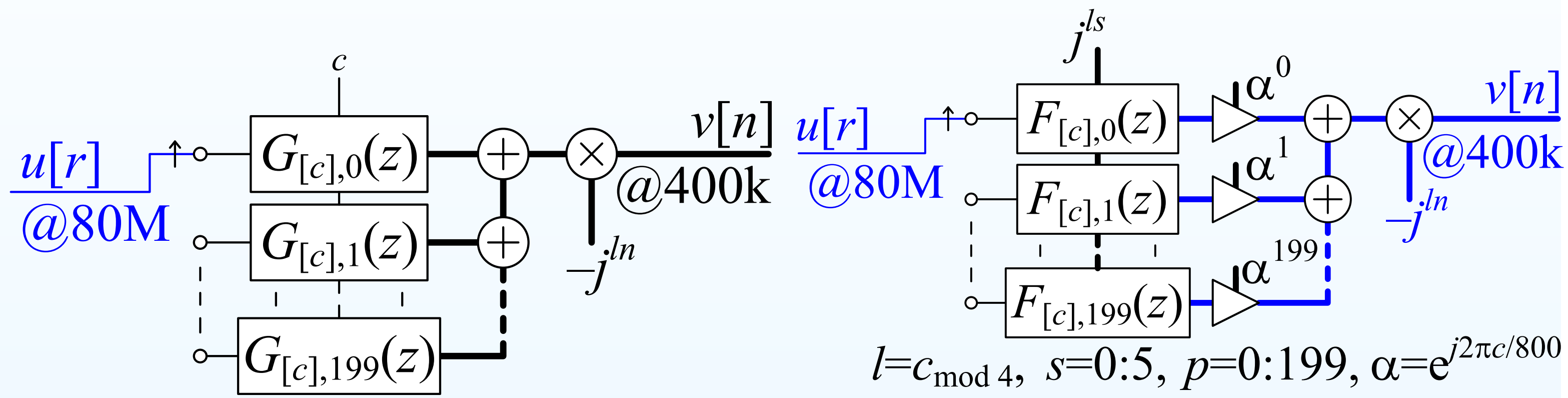

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Define $f_{[c],p}[s] = h[200s + p]e^{j2\pi\frac{(4k+l)s}{4}} = j^{ls}h[200s + p]$

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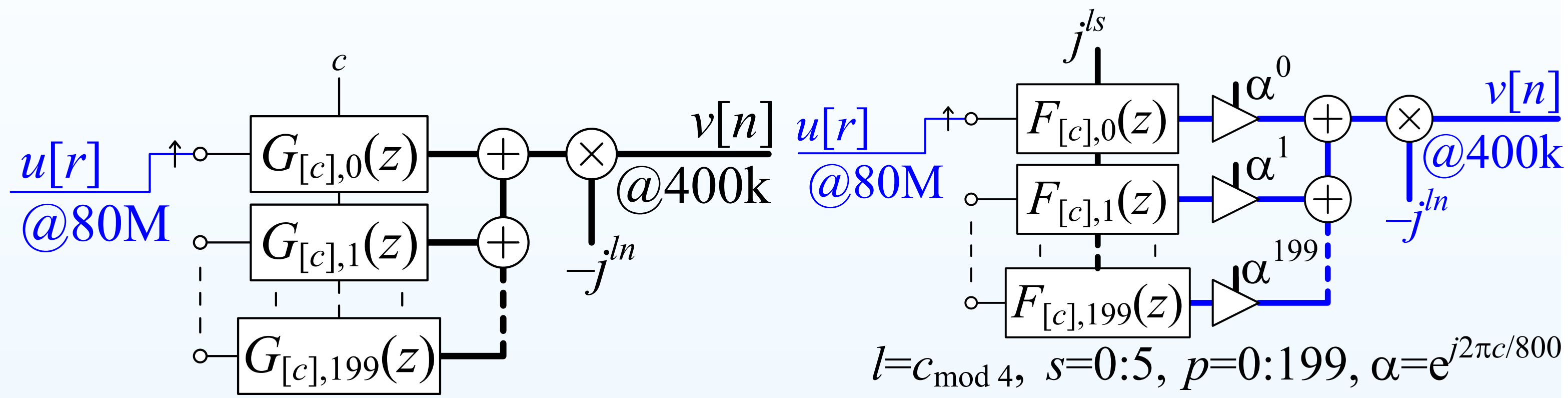


$$g_{[c]}[m] = h[m]e^{j2\pi\frac{cm}{800}}$$

$$g_{[c],p}[s] = g_c[200s+p] = h[200s+p]e^{j2\pi\frac{c(200s+p)}{800}}$$ [polyphase]

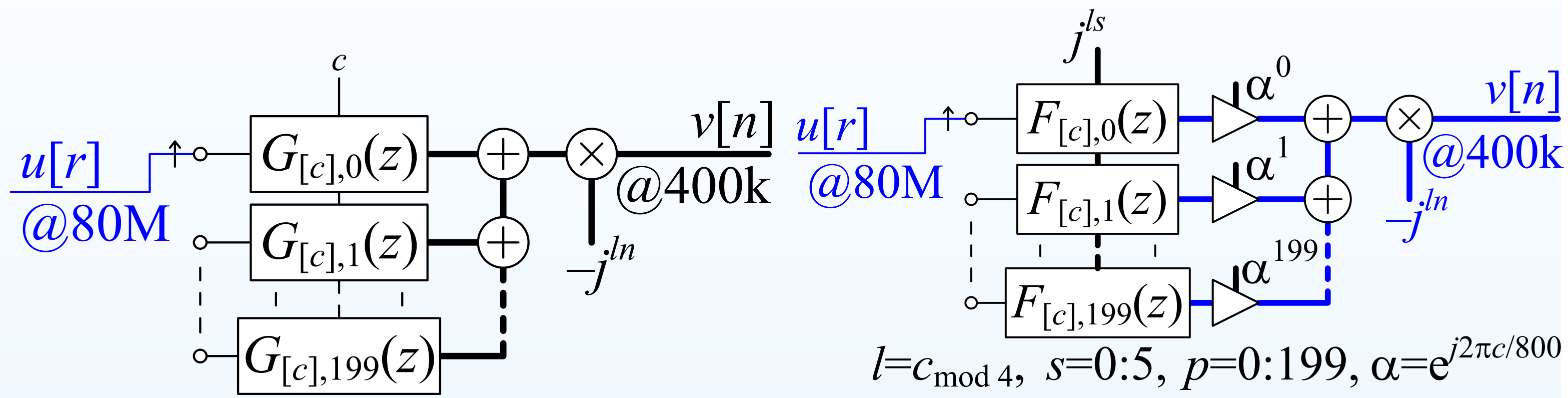
$$= h[200s+p]e^{j2\pi\frac{cs}{4}}e^{j2\pi\frac{cp}{800}} \triangleq h[200s+p]e^{j2\pi\frac{cs}{4}}\alpha^p$$

Define $f_{[c],p}[s] = h[200s+p]e^{j2\pi\frac{(4k+l)s}{4}} = j^{ls}h[200s+p]$

Although $f_{[c],p}[s]$ is complex it requires only one multiplication per tap because each tap is either purely real or purely imaginary.

Channel Selection (3)

Channel frequency $f_c = c \times 100\,\mathrm{kHz}$ where $c = 4k + l$ is an integer



$$g_{[c]}[m] = h[m]e^{j2\pi\frac{cm}{800}}$$

$$g_{[c],p}[s] = g_c[200s + p] = h[200s + p]e^{j2\pi\frac{c(200s+p)}{800}}$$ [polyphase]

$$= h[200s + p]e^{j2\pi\frac{cs}{4}}e^{j2\pi\frac{cp}{800}} \triangleq h[200s + p]e^{j2\pi\frac{cs}{4}}\alpha^p$$

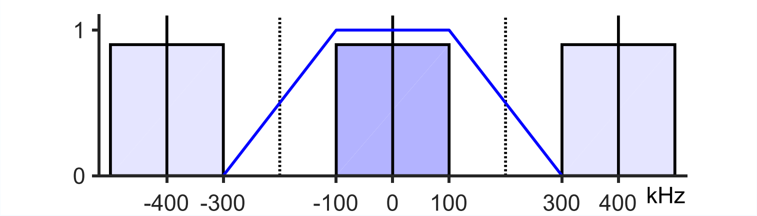
Define $f_{[c],p}[s] = h[200s + p]e^{j2\pi\frac{(4k+l)s}{4}} = j^{ls}h[200s + p]$

Although $f_{[c],p}[s]$ is complex it requires only one multiplication per tap because each tap is either purely real or purely imaginary.

Multiplication Load:

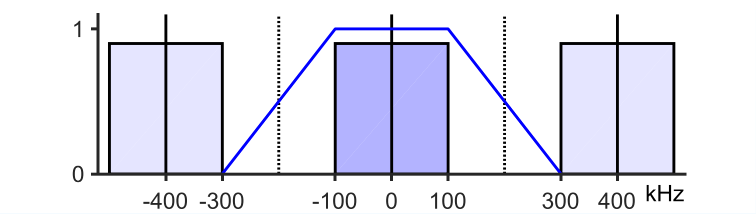
6×80 MHz $(F_p(z))$ + $4 \times 80\,\mathrm{MHz}$ $(\times e^{j2\pi\frac{cp}{800}})$ = $10 \times 80\,\mathrm{MHz}$

FM Demodulator



Complex FM signal centred at DC: $v(t) = |v(t)|e^{j\phi(t)}$

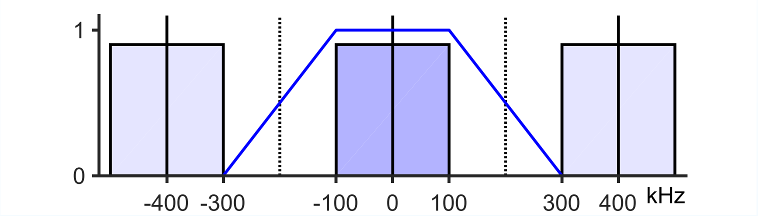
FM Demodulator



Complex FM signal centred at DC: $v(t) = |v(t)|e^{j\phi(t)}$

We know that $\log v = \log|v| + j\phi$

FM Demodulator

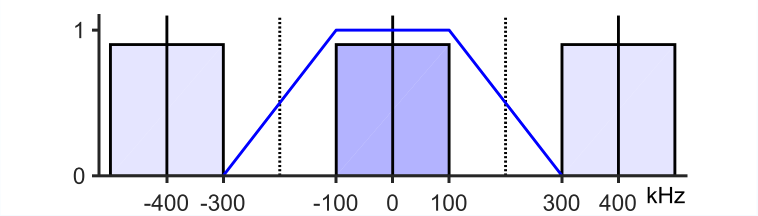


Complex FM signal centred at DC: $v(t) = |v(t)|e^{j\phi(t)}$

We know that $\log v = \log|v| + j\phi$

The instantaneous frequency of $v(t)$ is $\frac{d\phi}{dt}$.

FM Demodulator



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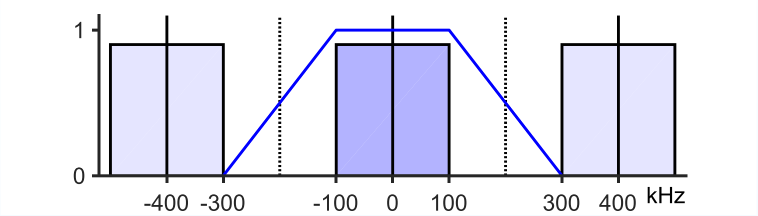
The instantaneous frequency of $v(t)$ is $\frac{d\phi}{dt}$.

We need to calculate $x(t) = \frac{d\phi}{dt} = \frac{d\Im(\log v)}{dt}$

FM Demodulator



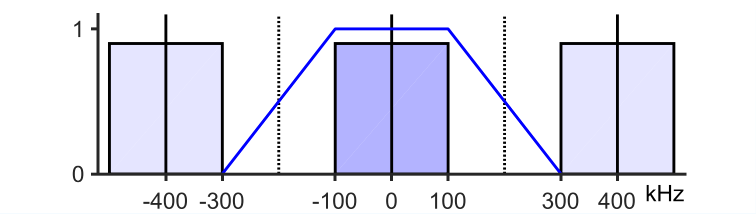
Complex FM signal centred at DC: $v(t) = |v(t)|e^{j\phi(t)}$

We know that $\log v = \log |v| + j\phi$

The instantaneous frequency of $v(t)$ is $\frac{d\phi}{dt}$.

We need to calculate $x(t) = \frac{d\phi}{dt} = \frac{d\Im(\log v)}{dt} = \Im\left(\frac{1}{v}\frac{dv}{dt}\right)$

FM Demodulator



Complex FM signal centred at DC: $v(t) = |v(t)|e^{j\phi(t)}$

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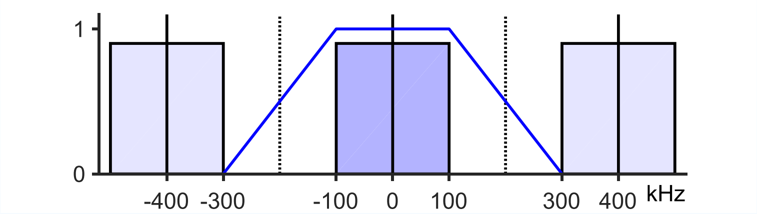
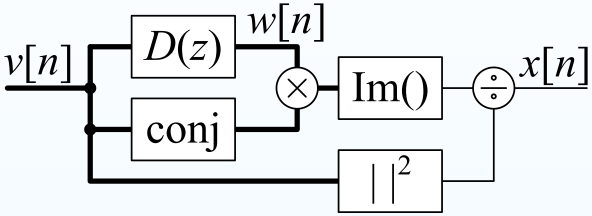
We need to calculate $x(t) = \frac{d\phi}{dt} = \frac{d\Im(\log v)}{dt} = \Im\left(\frac{1}{v}\frac{dv}{dt}\right) = \frac{1}{|v|^2}\Im\left(v^*\frac{dv}{dt}\right)$

FM Demodulator





Complex FM signal centred at DC: $v(t) = |v(t)|e^{j\phi(t)}$

We know that $\log v = \log |v| + j\phi$

The instantaneous frequency of $v(t)$ is $\frac{d\phi}{dt}$.

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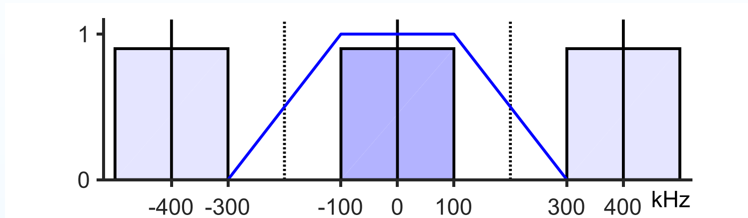
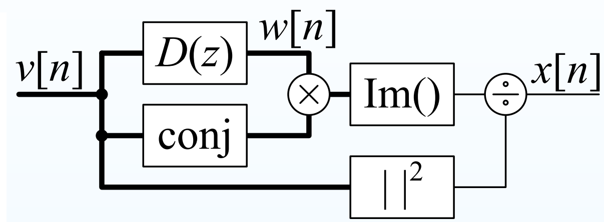
FM Demodulator





Complex FM signal centred at DC: $v(t) = |v(t)|e^{j\phi(t)}$

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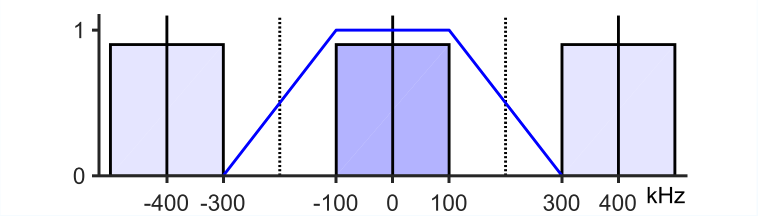
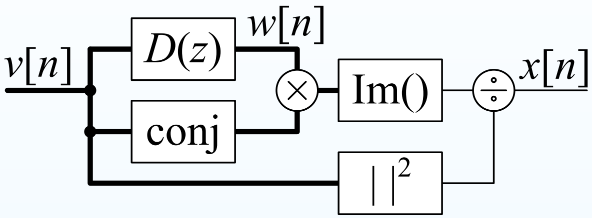
We need:

(1) Differentiation filter, $D(z)$

FM Demodulator





Complex FM signal centred at DC: $v(t) = |v(t)|e^{j\phi(t)}$

We know that $\log v = \log|v| + j\phi$

The instantaneous frequency of $v(t)$ is $\frac{d\phi}{dt}$.

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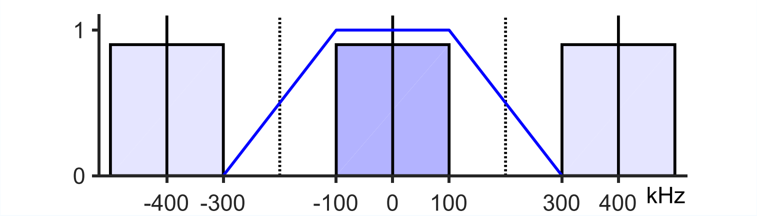
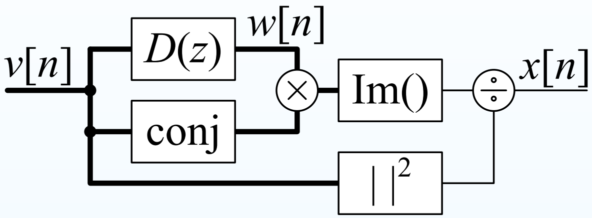
We need:

(1) Differentiation filter, $D(z)$

(2) Complex multiply, $w[n] \times v^*[n]$ (only need $\Im$ part)

FM Demodulator





Complex FM signal centred at DC: $v(t) = |v(t)|e^{j\phi(t)}$

We know that $\log v = \log|v| + j\phi$

The instantaneous frequency of $v(t)$ is $\frac{d\phi}{dt}$.

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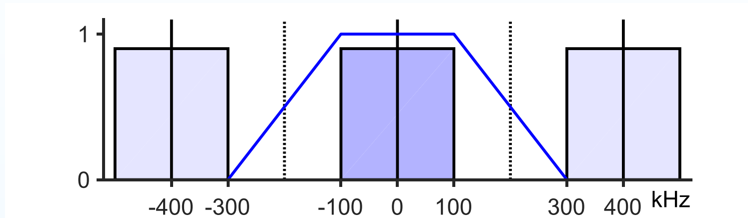
We need:

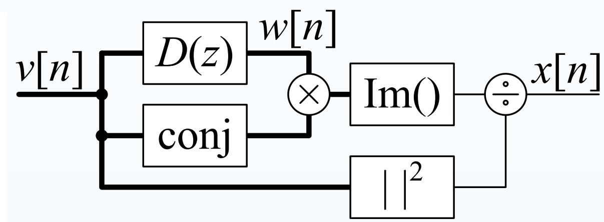
(1) Differentiation filter, $D(z)$

(2) Complex multiply, $w[n] \times v^*[n]$ (only need $\Im$ part)

(3) Real Divide by $|v|^2$

FM Demodulator





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We know that $\log v = \log|v| + j\phi$

The instantaneous frequency of $v(t)$ is $\frac{d\phi}{dt}$.

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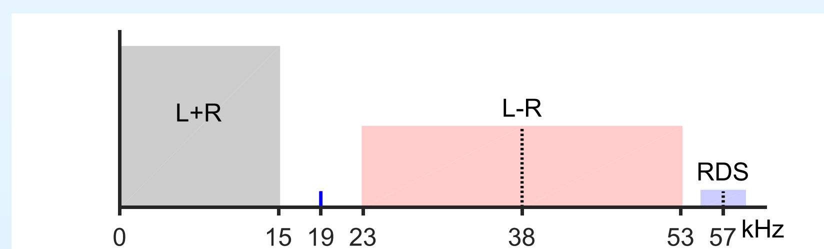
We need:

(1) Differentiation filter, $D(z)$

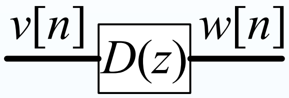
(2) Complex multiply, $w[n] \times v^*[n]$ (only need $\Im$ part)

(3) Real Divide by $|v|^2$

$x[n]$ is baseband signal (real):

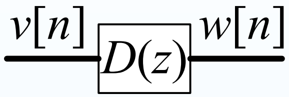


Differentiation Filter



Differentiation Filter

Window design method:
(1) calculate $d[n]$ for the ideal filter
(2) multiply by a window to give finite support



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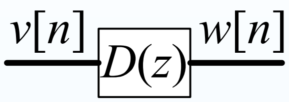

Differentiation: $\frac{d}{dt}e^{j\omega t} = j\omega e^{j\omega t}$

Differentiation Filter

Window design method:
(1) calculate $d[n]$ for the ideal filter
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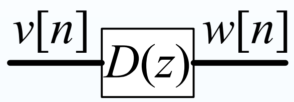

Differentiation: $\frac{d}{dt}e^{j\omega t} = j\omega e^{j\omega t} \quad \Rightarrow \quad D(e^{j\omega}) = \begin{cases} j\omega & |\omega| \le \omega_0 \\ 0 & |\omega| > \omega_0 \end{cases}$

Differentiation Filter

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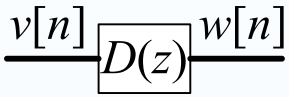

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Hence $d[n] = \frac{1}{2\pi}\int_{-\omega_0}^{\omega_0} j\omega e^{j\omega n} d\omega = \frac{j}{2\pi}\left[\frac{\omega e^{jn\omega}}{jn} - \frac{e^{jn\omega}}{j^2 n^2}\right]_{-\omega_0}^{\omega_0}$ [IDTFT]

$= \frac{n\omega_0 \cos n\omega_0 - \sin n\omega_0}{\pi n^2}$

Differentiation Filter

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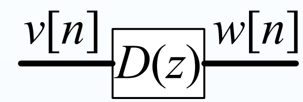

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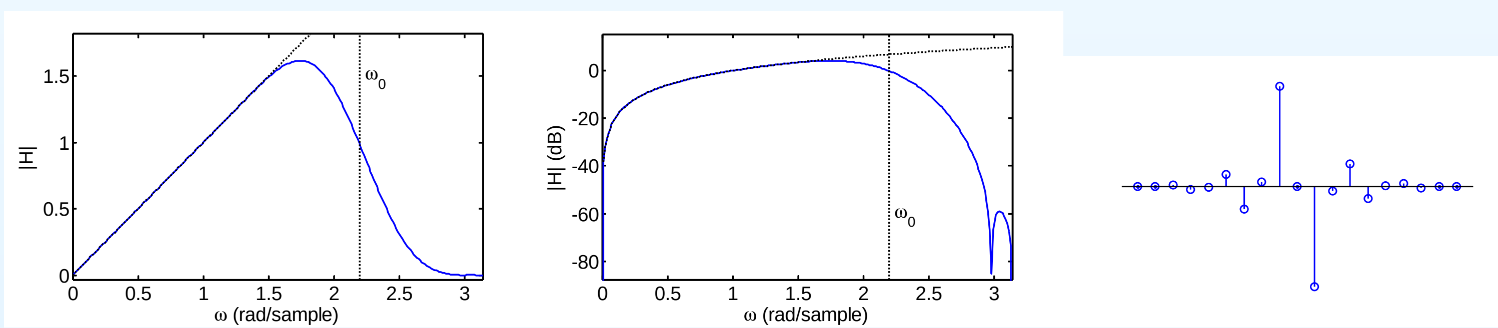

Using $M = 18$, Kaiser window, $\beta = 7$ and $\omega_0 = 2.2 = \frac{2\pi \times 140\,\text{kHz}}{400\,\text{kHz}}$:

Differentiation Filter

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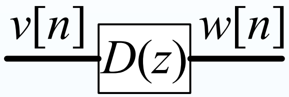

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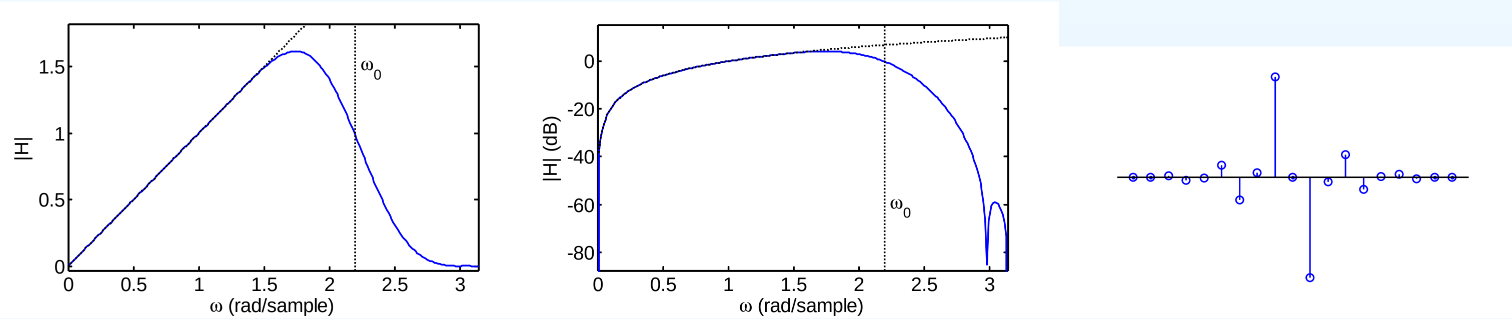

Using $M = 18$, Kaiser window, $\beta = 7$ and $\omega_0 = 2.2 = \frac{2\pi \times 140\,\text{kHz}}{400\,\text{kHz}}$:
Near perfect differentiation for $\omega \le 1.6$ ($\approx 100\,\text{kHz}$ for $f_s = 400\,\text{kHz}$)

Differentiation Filter

Window design method:

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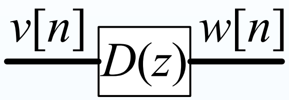

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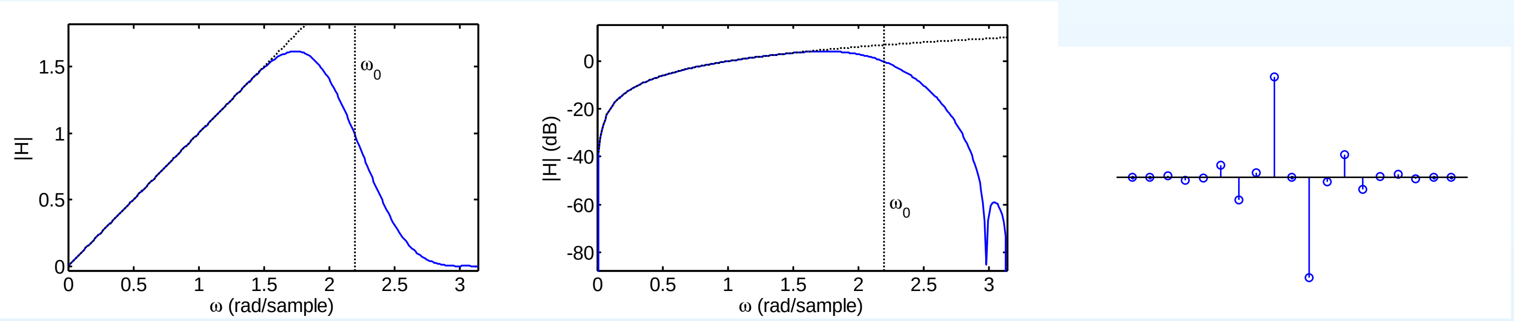


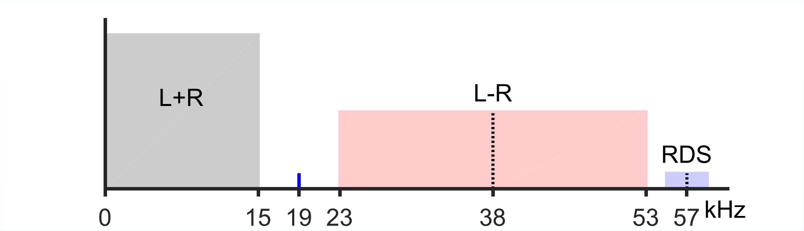
Using $M = 18$, Kaiser window, $\beta = 7$ and $\omega_0 = 2.2 = \frac{2\pi \times 140\,\text{kHz}}{400\,\text{kHz}}$:

Near perfect differentiation for $\omega \le 1.6$ ($\approx 100\,\text{kHz}$ for $f_s = 400\,\text{kHz}$)

Broad transition region allows shorter filter

Pilot tone extraction

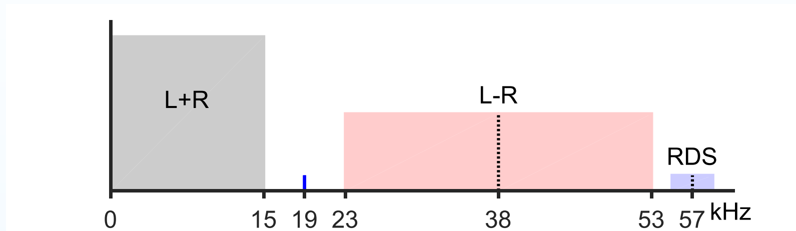
+

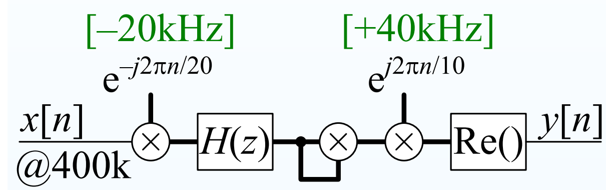


Aim: extract $19\,\mathrm{kHz}$ pilot tone, double freq $\rightarrow$ real $38\,\mathrm{kHz}$ tone.

Pilot tone extraction

+





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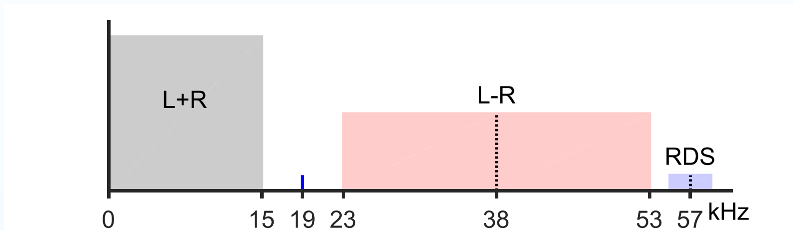
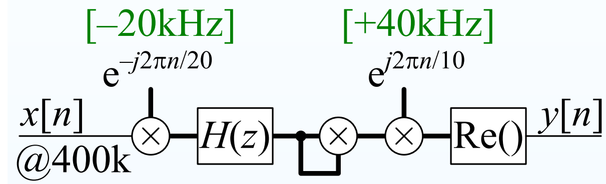
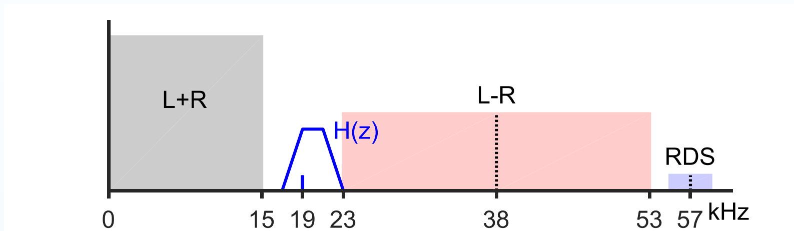


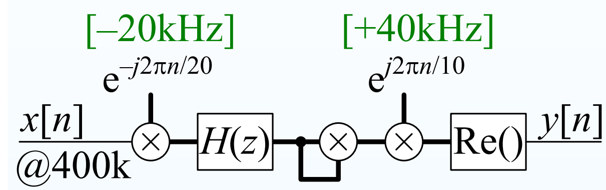


Aim: extract $19\,\text{kHz}$ pilot tone, double freq $\rightarrow$ real $38\,\text{kHz}$ tone.

(1) shift spectrum down by $20\,\text{kHz}$: multiply by $e^{-j2\pi n\frac{20\,\text{kHz}}{400\,\text{kHz}}}$

Pilot tone extraction





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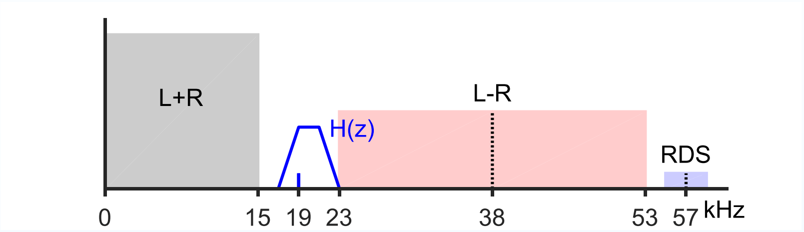
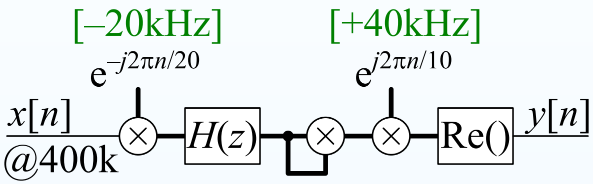
(2) low pass filter to $\pm 1\,\mathrm{kHz}$ to extract complex pilot at $-1\,\mathrm{kHz}$: $H(z)$

Pilot tone extraction





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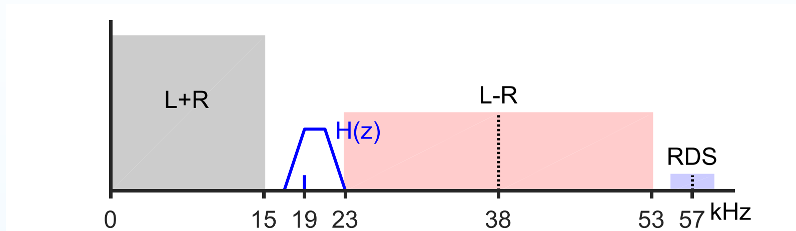
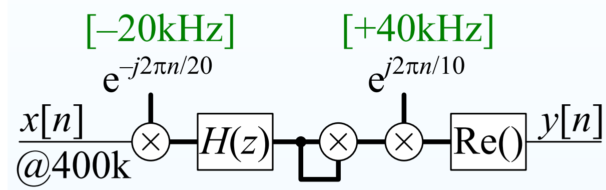
(2) low pass filter to $\pm 1\,\text{kHz}$ to extract complex pilot at $-1\,\text{kHz}$: $H(z)$

(3) square to double frequency to $-2\,\text{kHz}$ $\left[\left(e^{j\omega t}\right)^2 = e^{j2\omega t}\right]$

Pilot tone extraction





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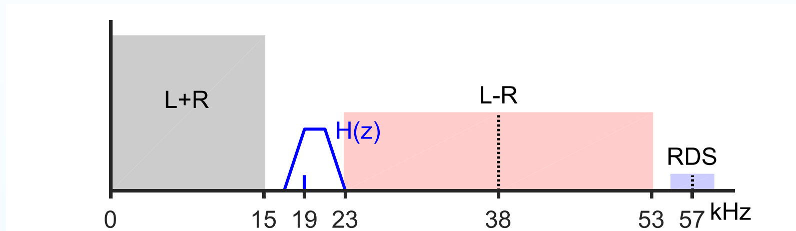
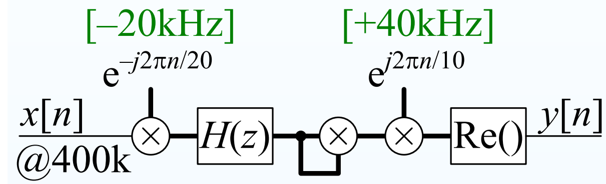
Pilot tone extraction





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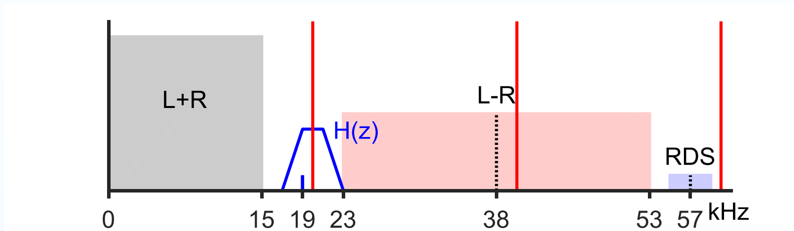
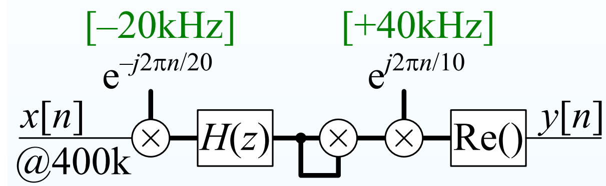
(5) take real part

Pilot tone extraction

+





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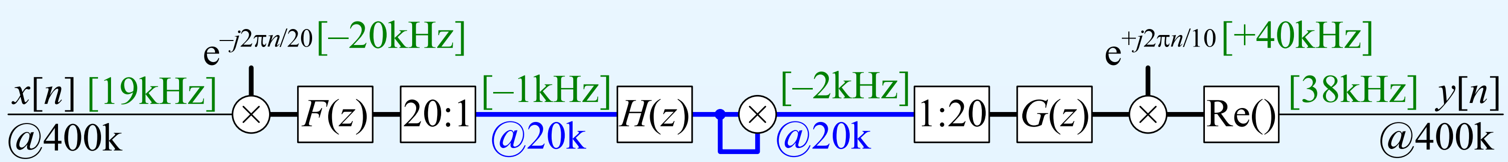

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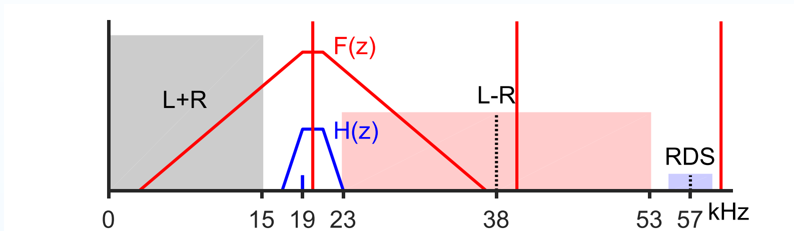

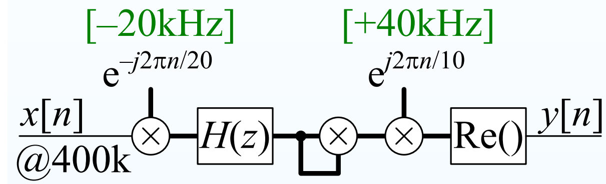

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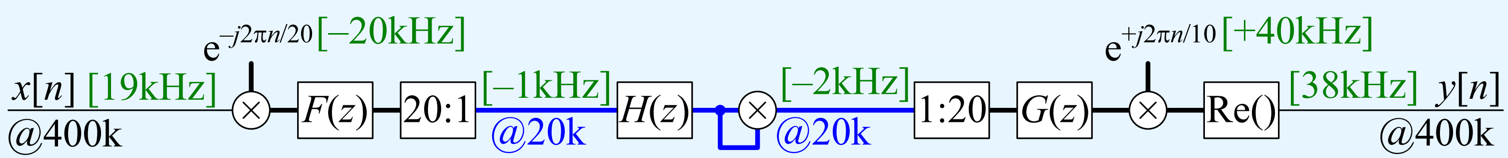
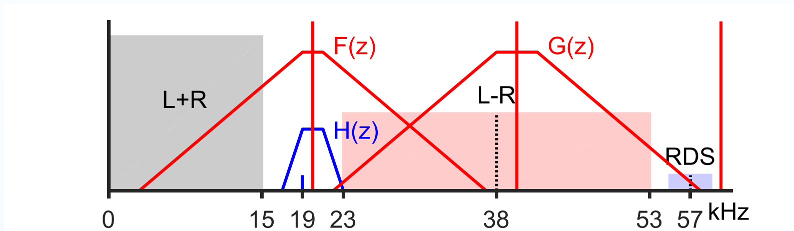
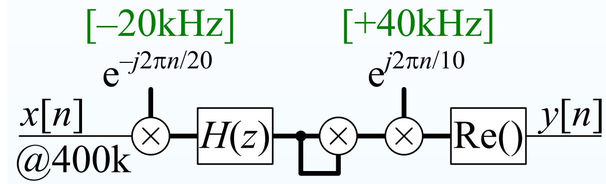


Transition bands:

$F(z)$: $1 \rightarrow 17\,\text{kHz}$, $\qquad H(z)$: $1 \rightarrow 3\,\text{kHz}$

Pilot tone extraction





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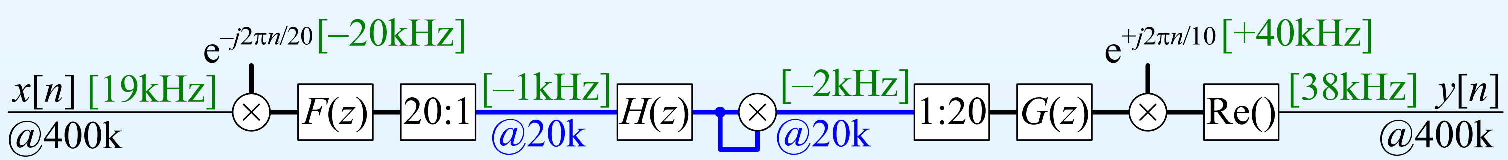

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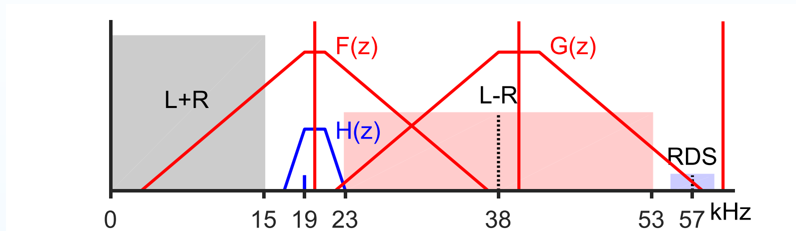
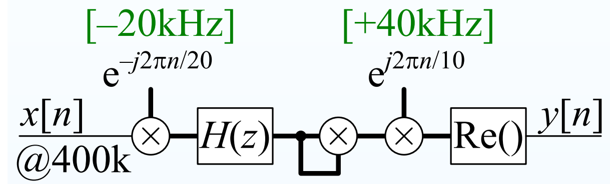
$F(z)$: $1 \rightarrow 17\,\text{kHz}$, $\quad H(z)$: $1 \rightarrow 3\,\text{kHz}$, $\quad G(z)$: $2 \rightarrow 18\,\text{kHz}$

Pilot tone extraction +





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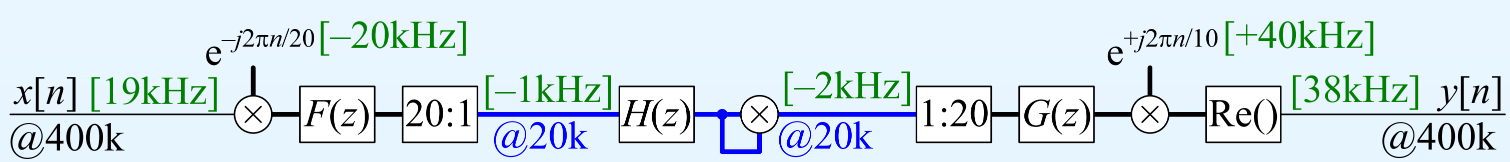

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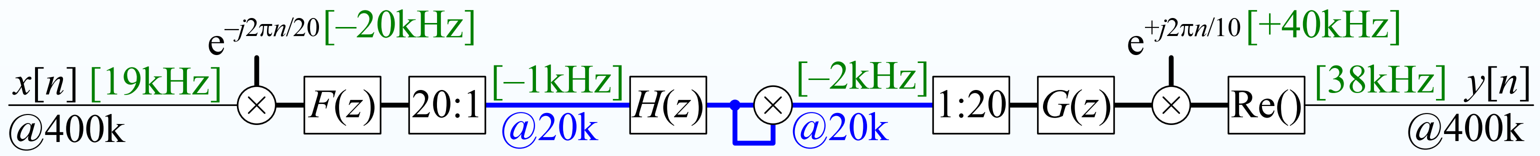
$\Delta\omega = 0.25 \Rightarrow M = 68$, $\Delta\omega = 0.63 \Rightarrow 27$, $\Delta\omega = 0.25 \Rightarrow 68$

Polyphase Pilot tone



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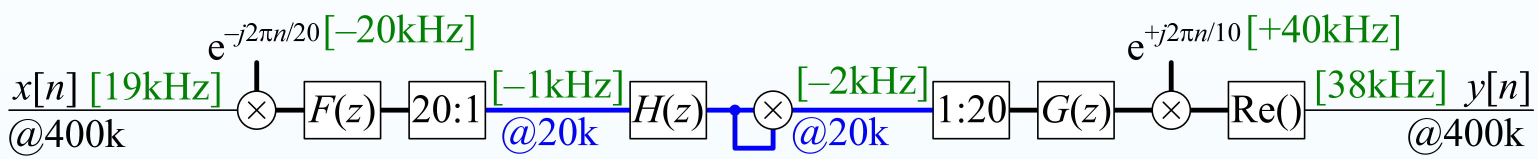

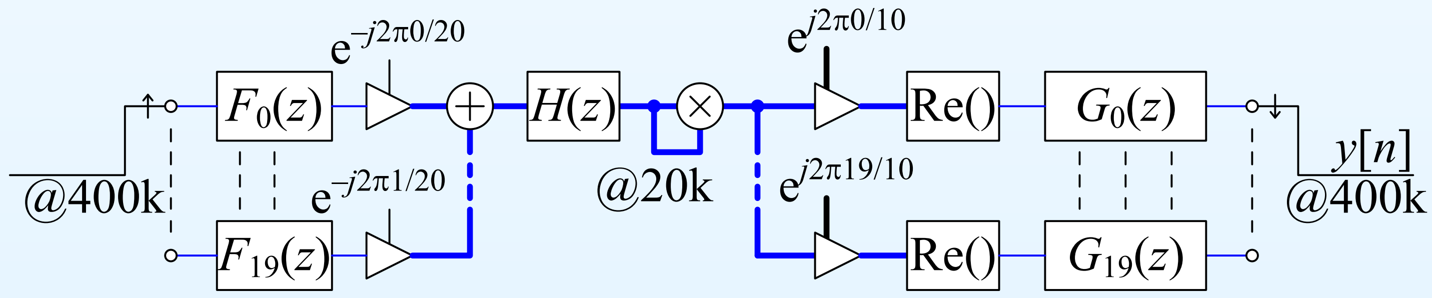

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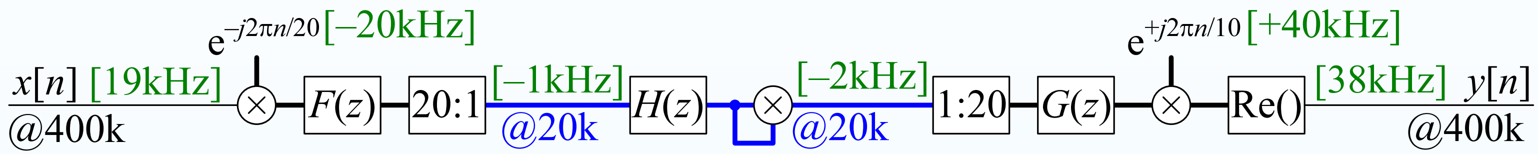
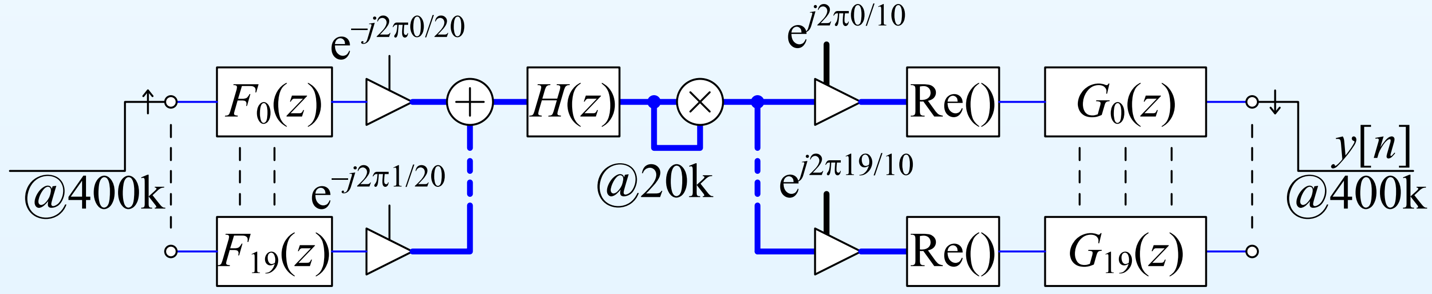


Anti-alias filter: $F(z)$

Each branch, $F_p(z)$, gets every 20^{th} sample and an identical $e^{j2\pi\frac{n}{20}}$



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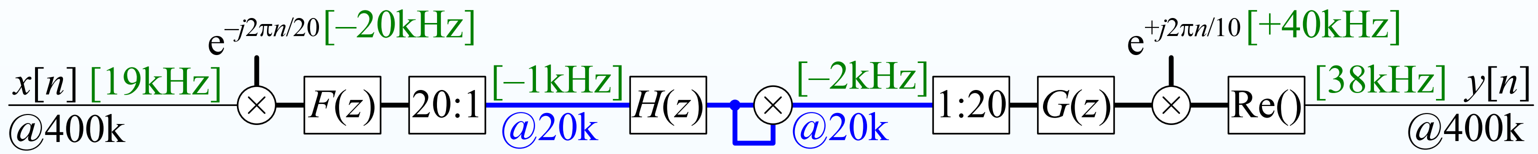

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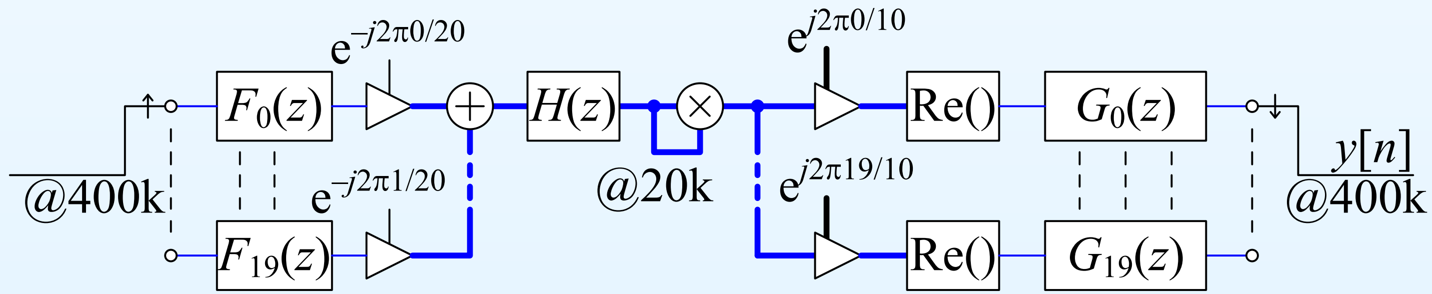
So $F_p(z)$ can filter a real signal and then multiply by fixed $e^{j2\pi\frac{p}{20}}$



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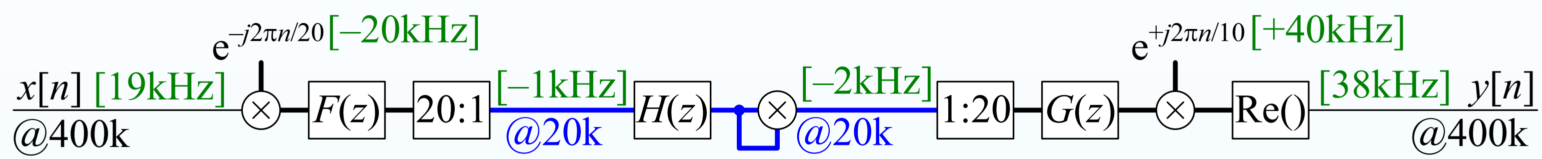

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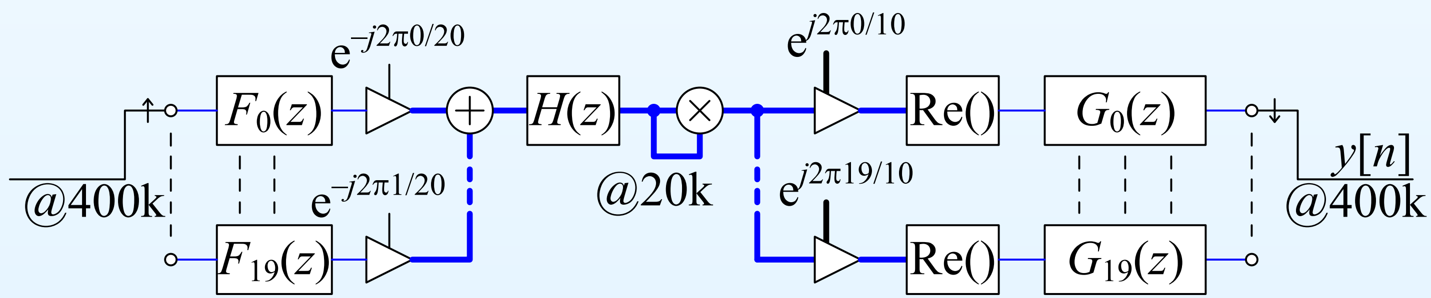

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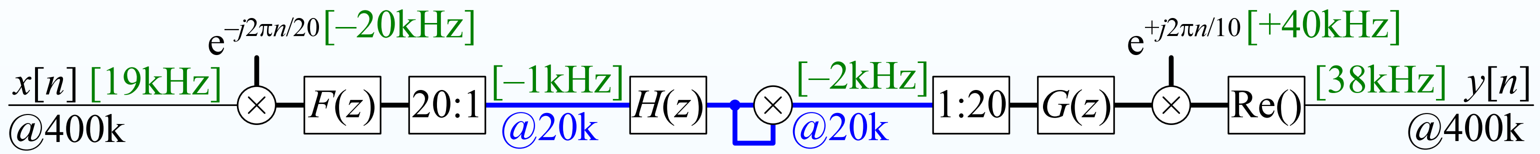

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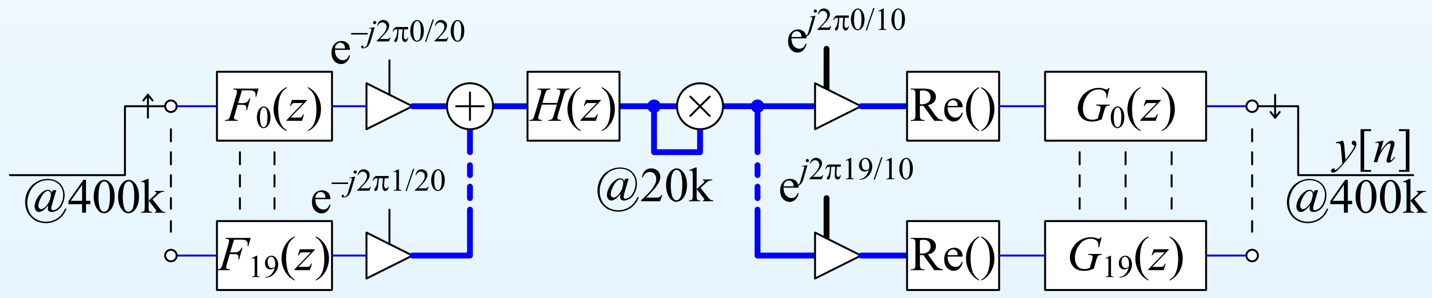

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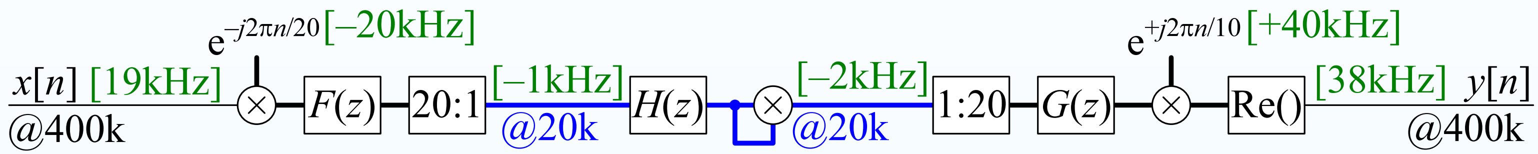

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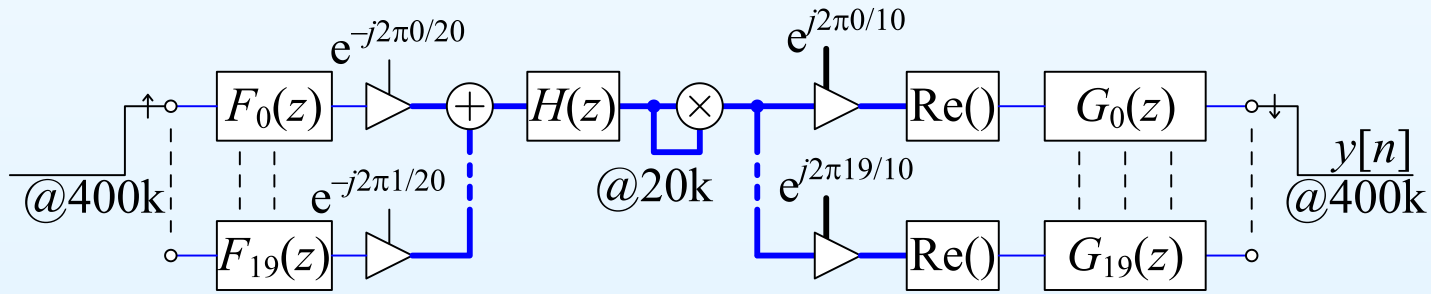

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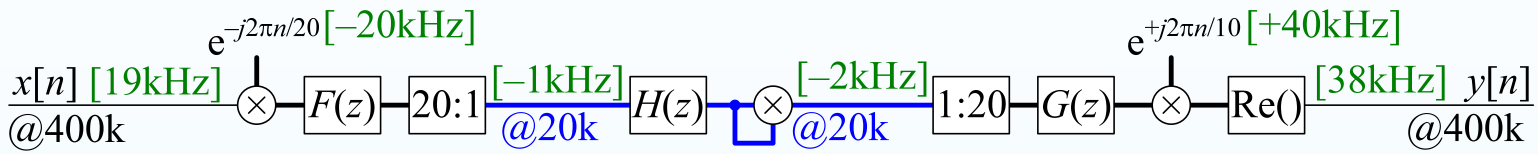
F and G each: $(4+2) \times 400\,\text{kHz}$

Polyphase Pilot tone



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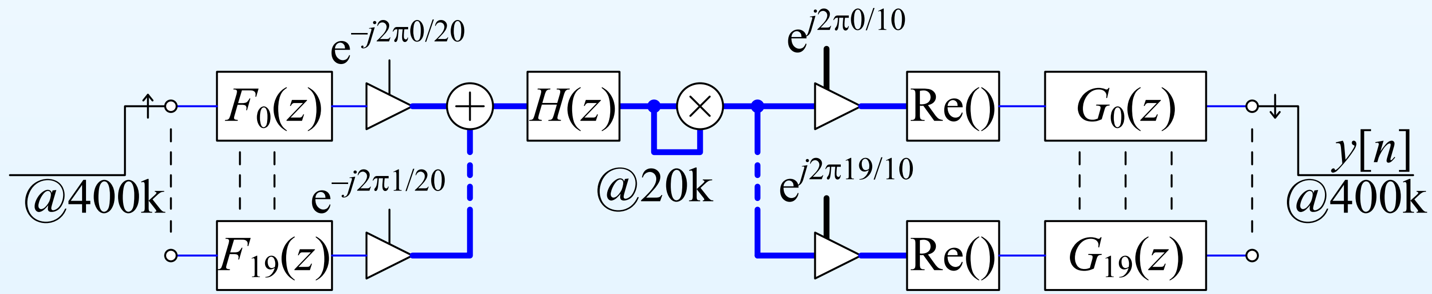

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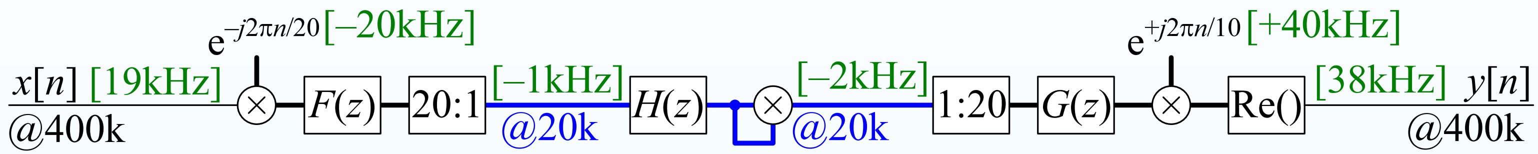
Polyphase Pilot tone



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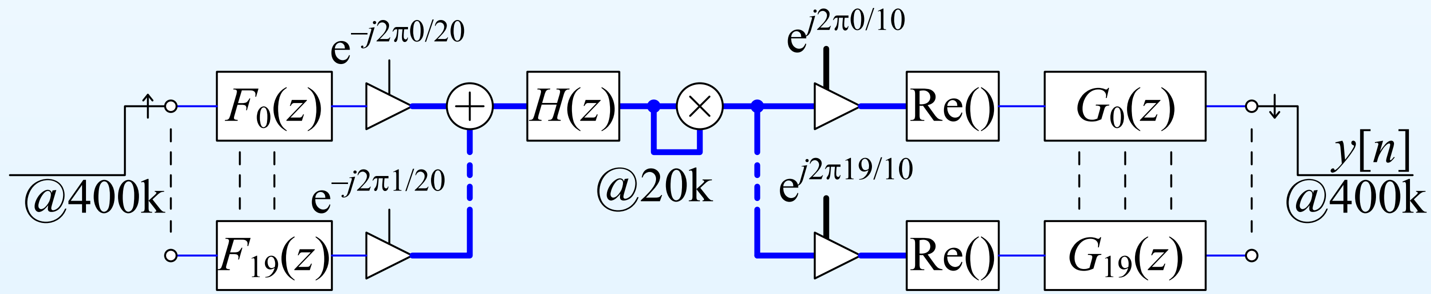

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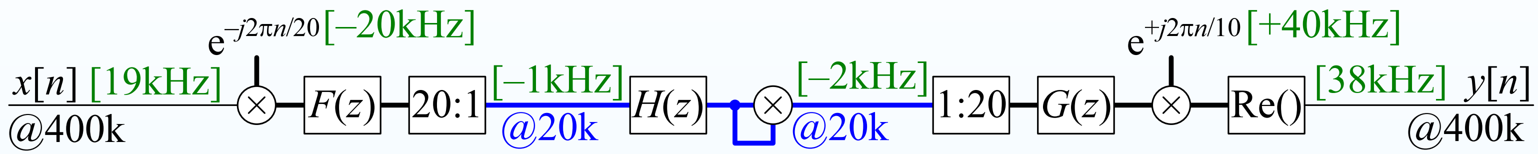
Total: $15\times 400\,\text{kHz}$

Polyphase Pilot tone



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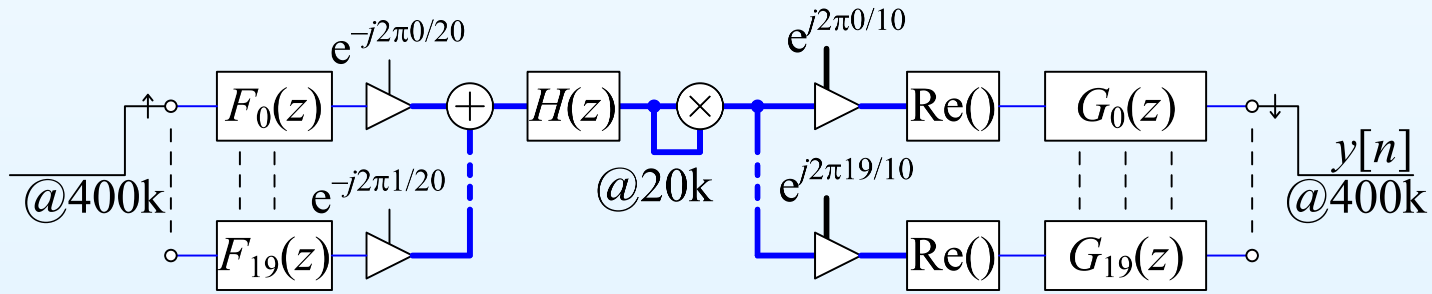

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This example is taken from Harris: 13.