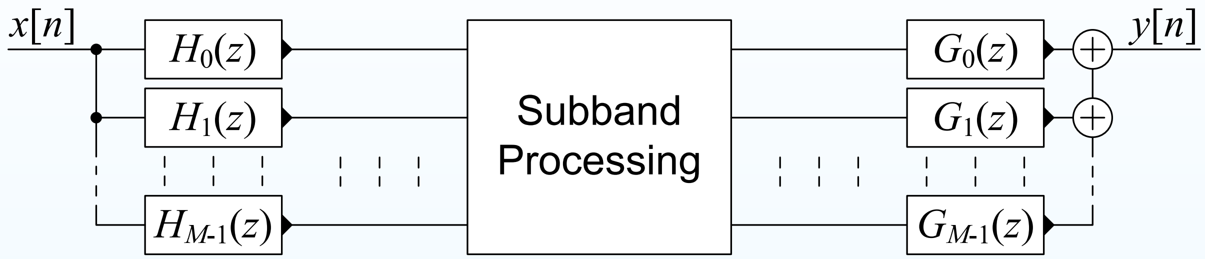
15: Subband Processing

Subband processing



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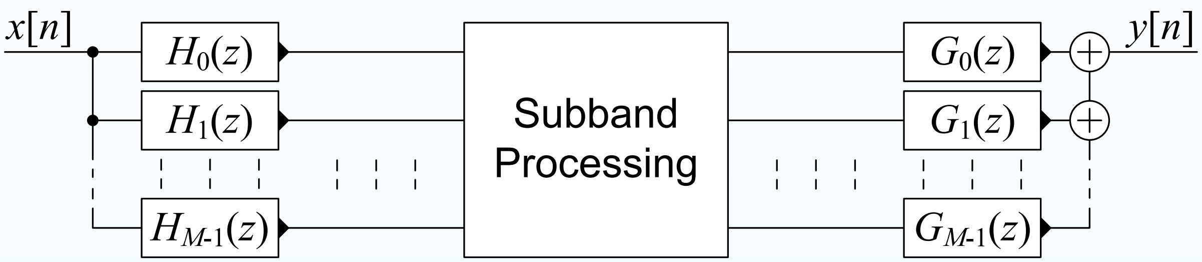

- The $H_m(z)$ are bandpass *analysis filters* and divide $x[n]$ into frequency bands

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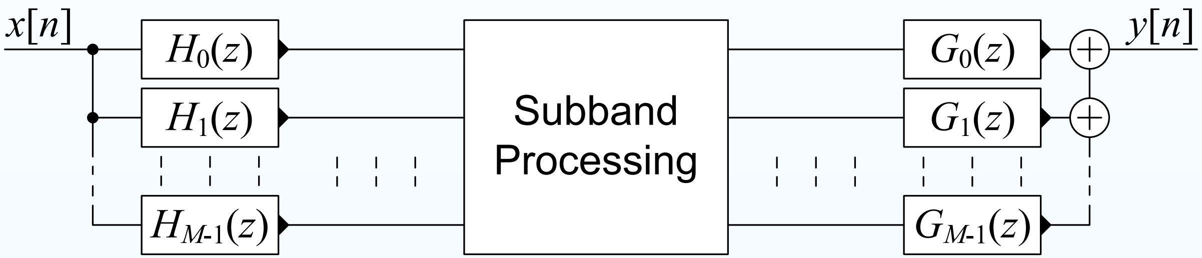

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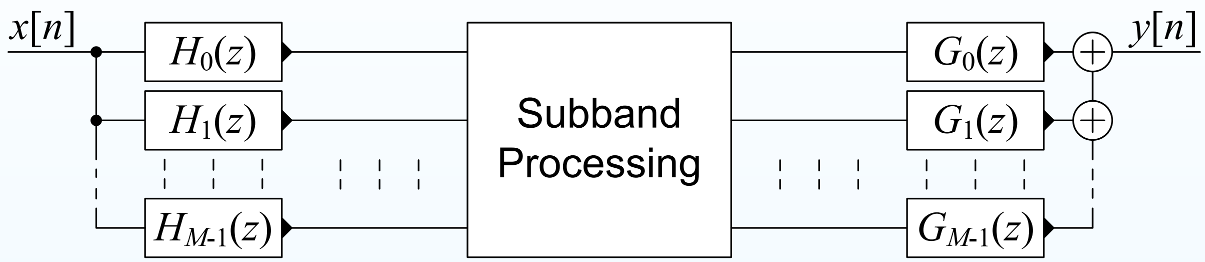

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- The $G_m(z)$ are *synthesis filters* and together reconstruct the output

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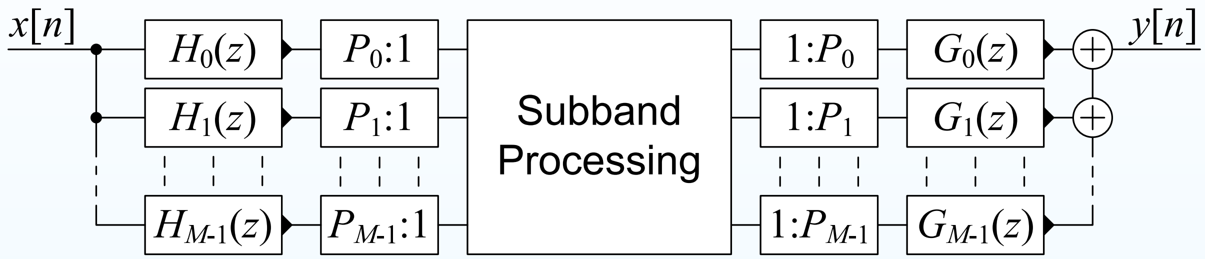

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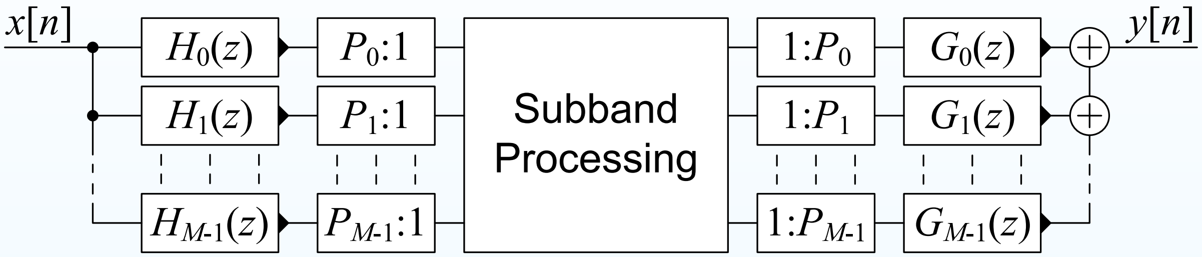

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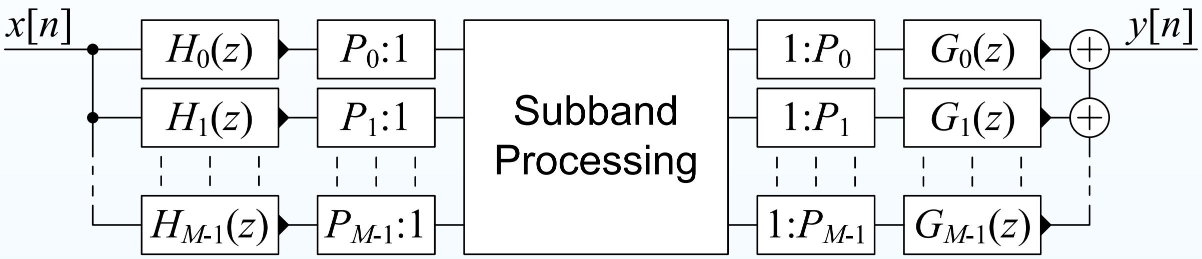

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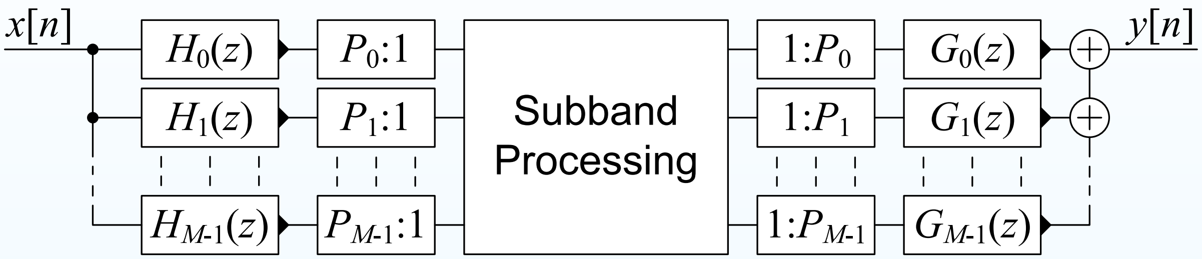

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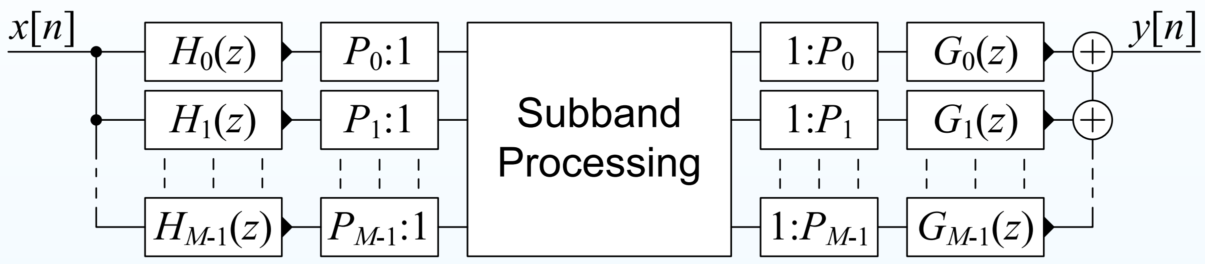

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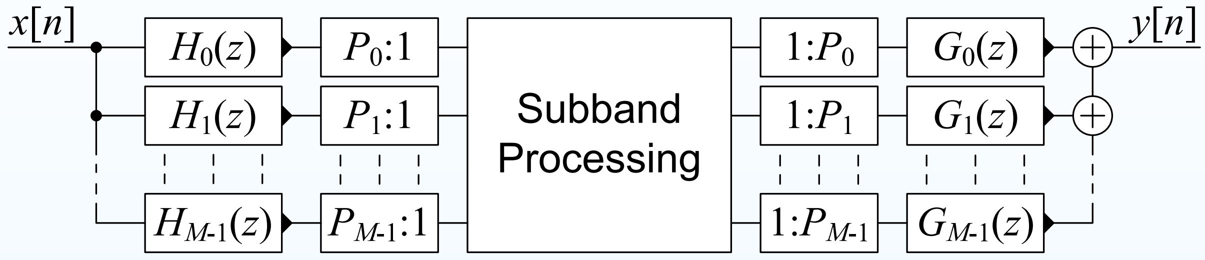

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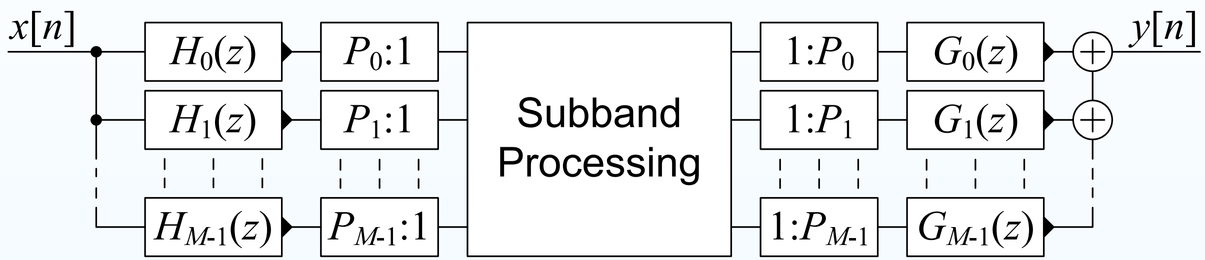

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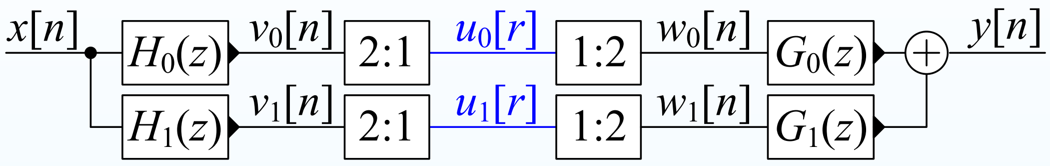
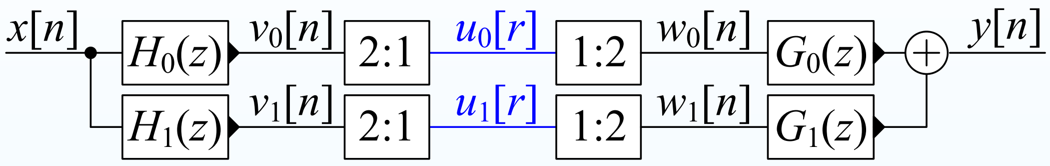
- Benefits: Lower computation, faster convergence if adaptive

2-band Filterbank



2-band Filterbank



$$V_m(z) = H_m(z)X(z) \qquad [m \in \{0, 1\}]$$

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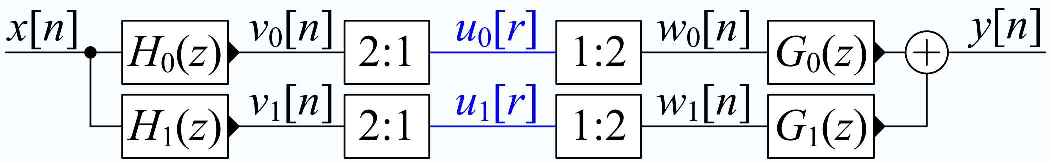

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$$U_m(z) = \frac{1}{K}\sum_{k=0}^{K-1} V_m(e^{\frac{-j2\pi k}{K}} z^{\frac{1}{K}})$$

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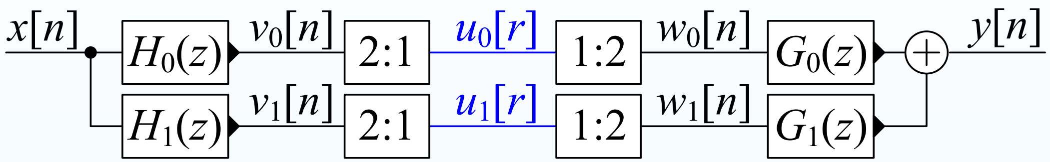

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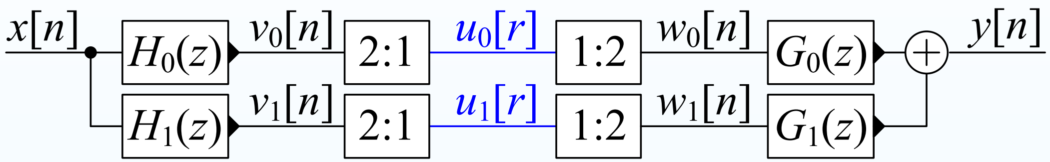

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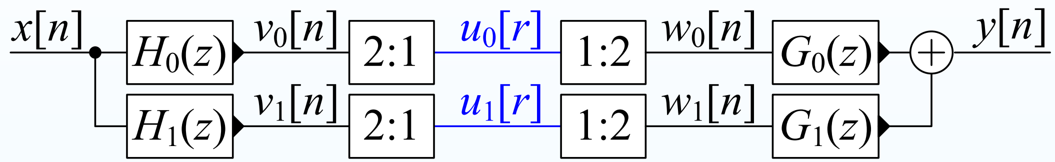

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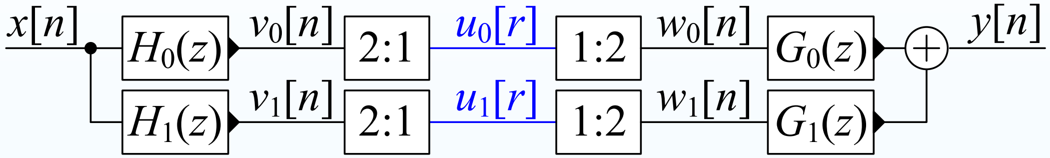

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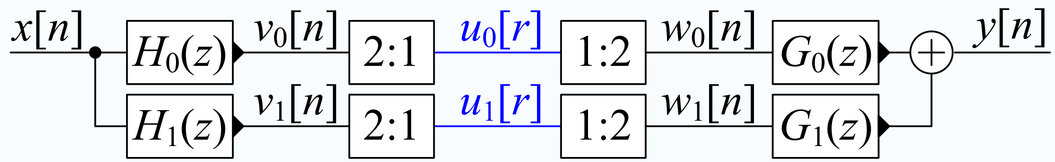

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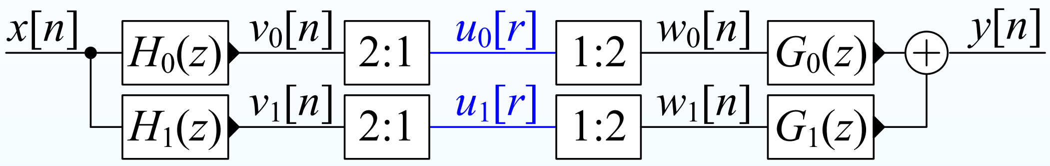

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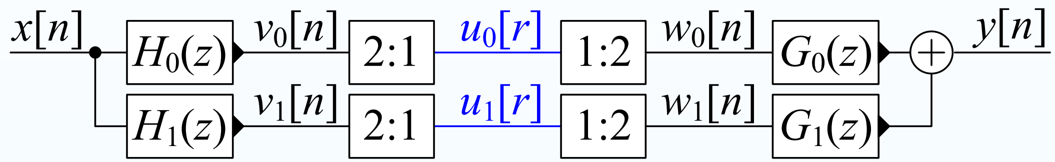

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We want (a) $T(z) = \frac{1}{2}\{H_0(z)G_0(z) + H_1(z)G_1(z)\} = z^{-d}$

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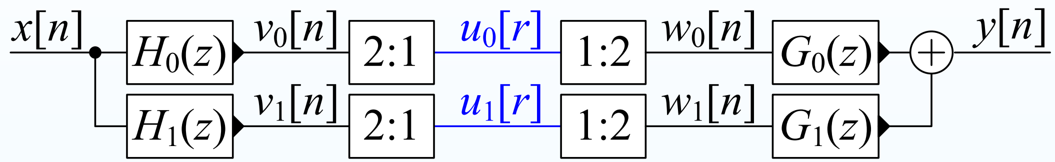

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$$= \begin{bmatrix} X(z) & X(-z) \end{bmatrix} \begin{bmatrix} T(z) \\ A(z) \end{bmatrix} \qquad [X(-z)A(z) \text{ is “aliased” term}]$$

We want (a) $T(z) = \frac{1}{2}\{H_0(z)G_0(z) + H_1(z)G_1(z)\} = z^{-d}$

and (b) $A(z) = \frac{1}{2}\{H_0(-z)G_0(z) + H_1(-z)G_1(z)\} = 0$

Perfect Reconstruction

For perfect reconstruction without aliasing, we require

$$\frac{1}{2}\begin{bmatrix} H_0(z) & H_1(z) \\ H_0(-z) & H_1(-z) \end{bmatrix}\begin{bmatrix} G_0(z) \\ G_1(z) \end{bmatrix} = \begin{bmatrix} z^{-d} \\ 0 \end{bmatrix}$$

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$$= \frac{2z^{-d}}{H_0(z)H_1(-z)-H_0(-z)H_1(z)}\begin{bmatrix} H_1(-z) & -H_1(z) \\ -H_0(-z) & H_0(z) \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

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For all filters to be FIR, we need the denominator to be

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$H_0(z)H_1(-z) - H_0(-z)H_1(z) = cz^{-k}$, which implies

$$\begin{bmatrix} G_0(z) \\ G_1(z) \end{bmatrix} = \frac{2}{c} z^{k-d}\begin{bmatrix} H_1(-z) \\ -H_0(-z) \end{bmatrix}$$

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For all filters to be FIR, we need the denominator to be

$H_0(z)H_1(-z) - H_0(-z)H_1(z) = cz^{-k}$, which implies

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Perfect Reconstruction

For perfect reconstruction without aliasing, we require

$$\frac{1}{2}\begin{bmatrix} H_0(z) & H_1(z) \\ H_0(-z) & H_1(-z) \end{bmatrix}\begin{bmatrix} G_0(z) \\ G_1(z) \end{bmatrix} = \begin{bmatrix} z^{-d} \\ 0 \end{bmatrix}$$

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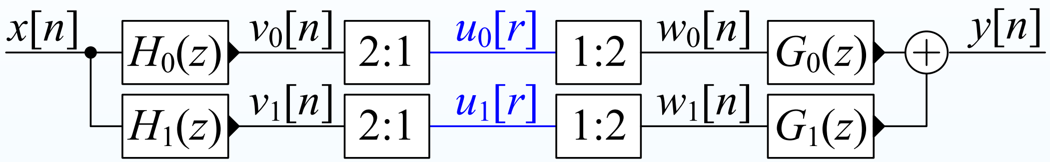
Note: c just scales $H_i(z)$ by $c^{\frac{1}{2}}$ and $G_i(z)$ by $c^{-\frac{1}{2}}$.

Quadrature Mirror Filterbank (QMF)



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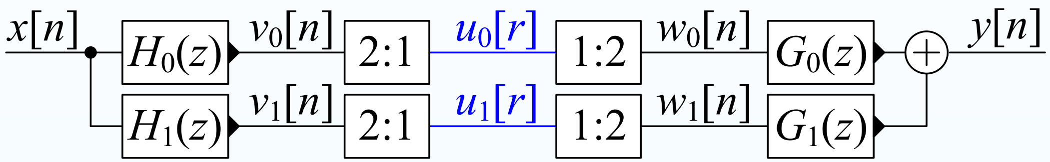

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(a) $H_0(z)$ is causal and real

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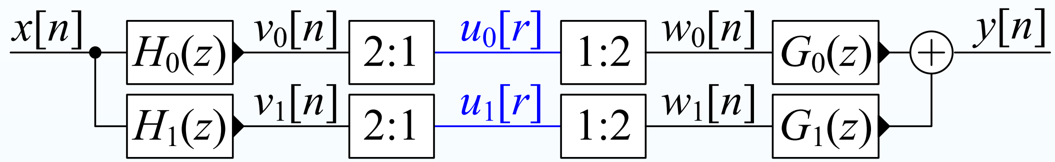

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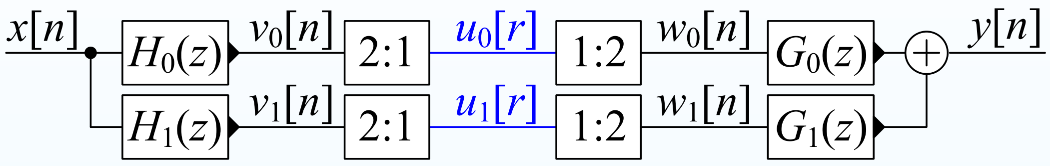

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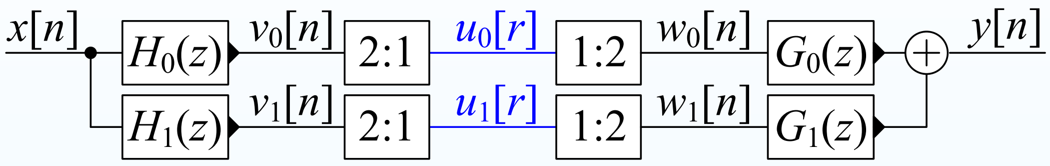

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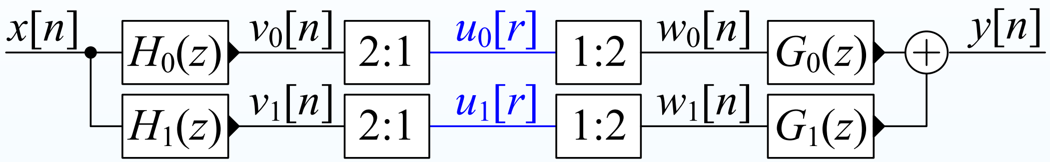

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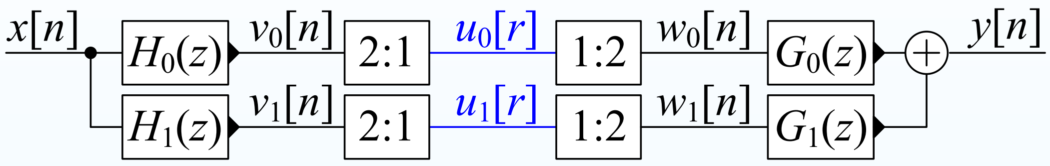

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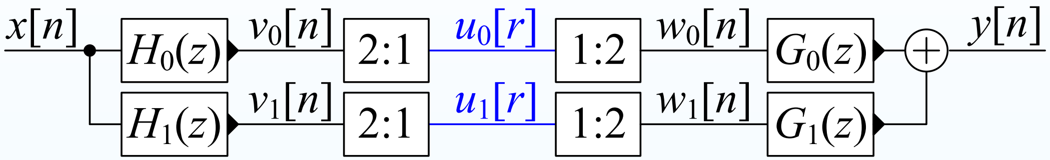

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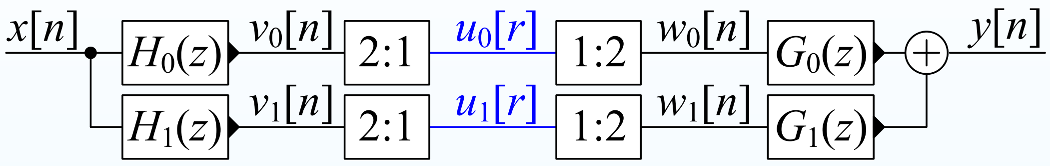

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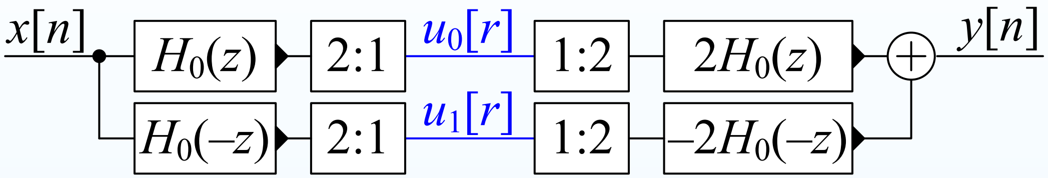
Polyphase QMF



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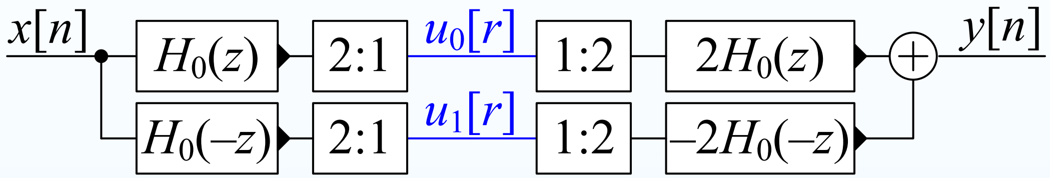

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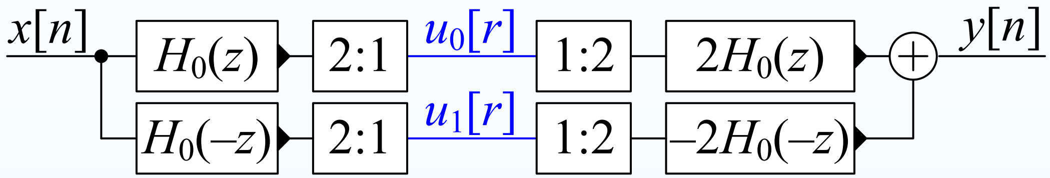

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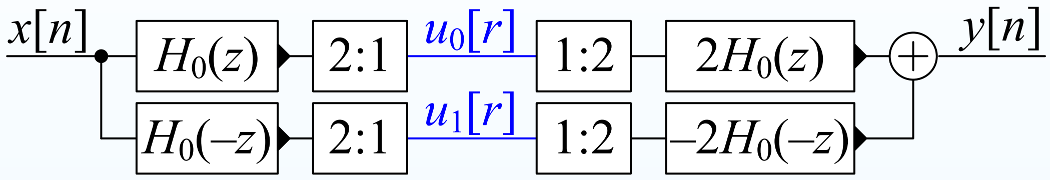

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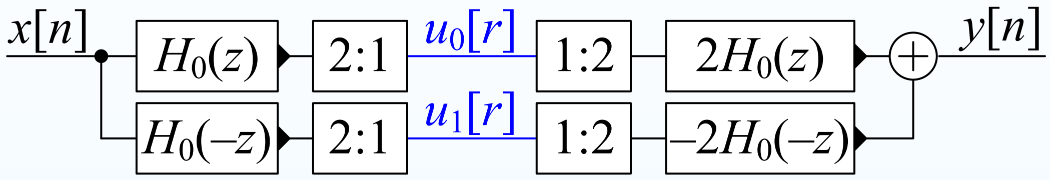

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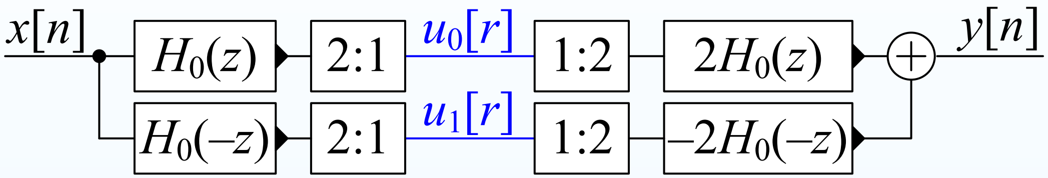

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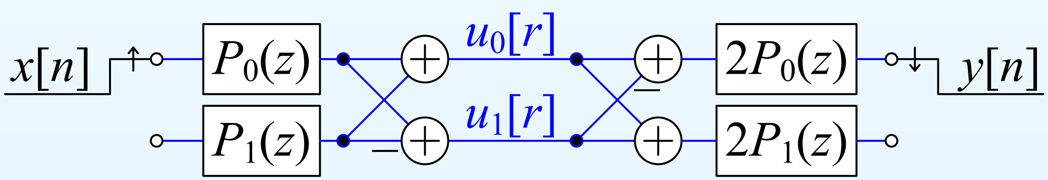

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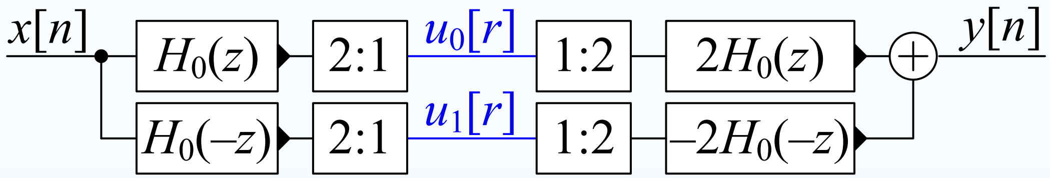

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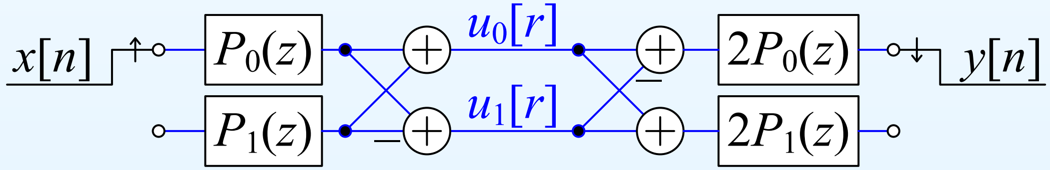

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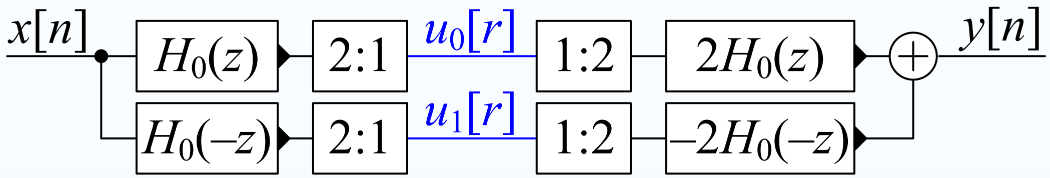

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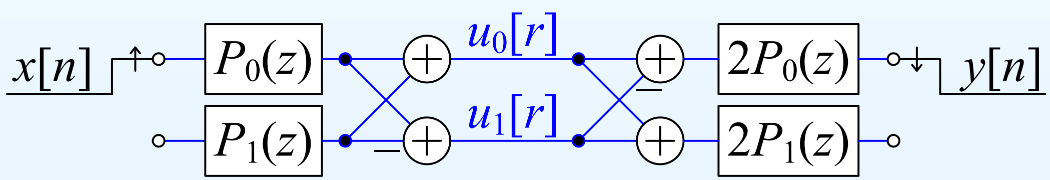

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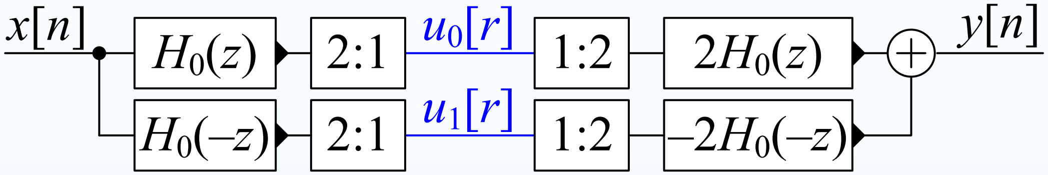

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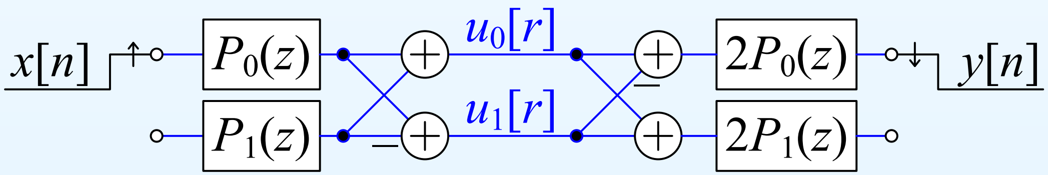

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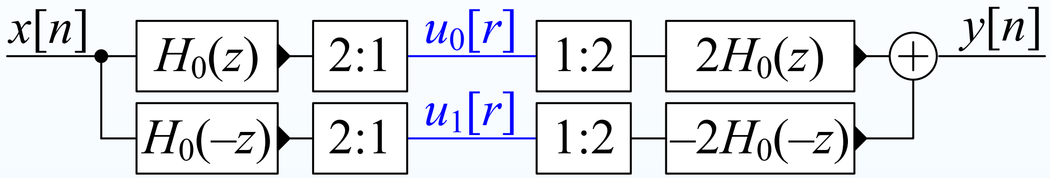

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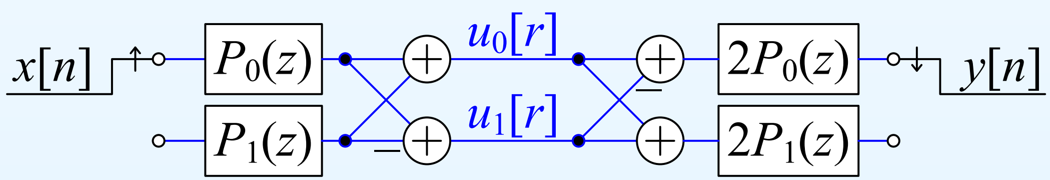

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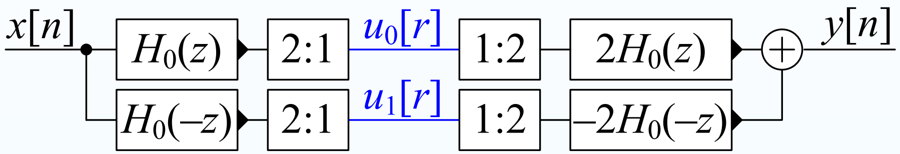
$\therefore$ Perfect reconstruction QMF filterbanks cannot have good freq selectivity

QMF Options



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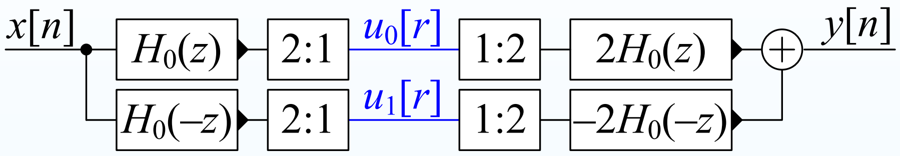

$A(z) = 0 \Rightarrow$ no alias term

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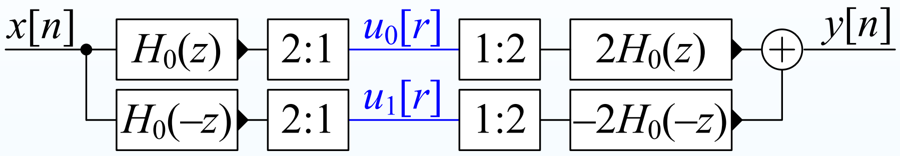

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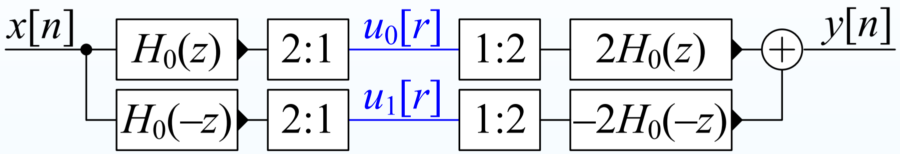

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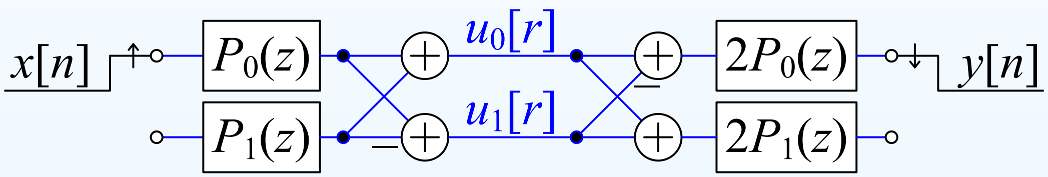

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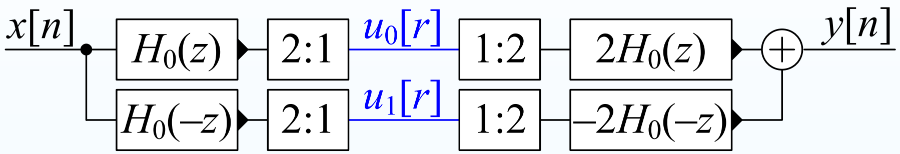

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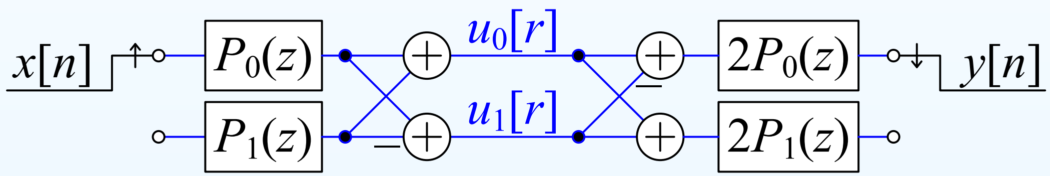

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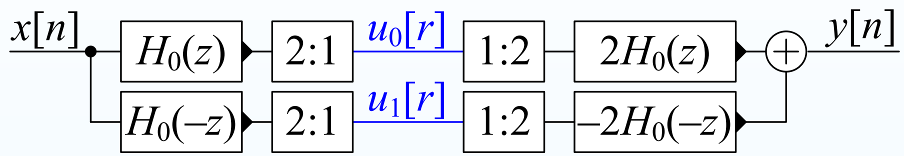

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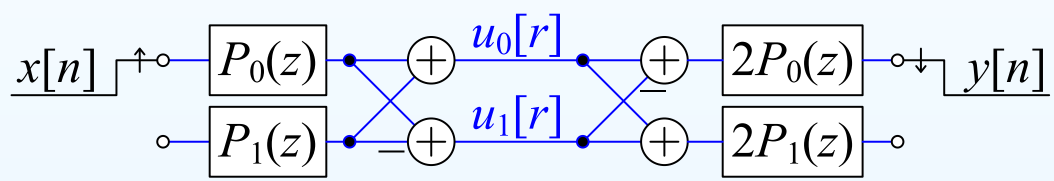

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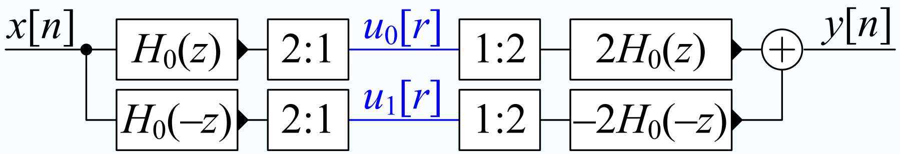
Options:

(A) Perfect Reconstruction: $T(z) = z^{-d} \Rightarrow H_0(z)$ is a bad filter.

QMF Options



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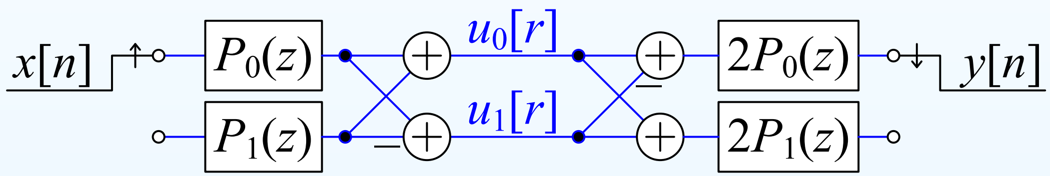

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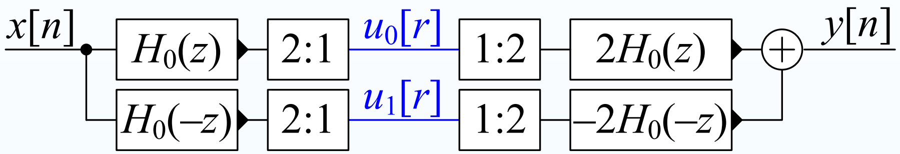
$T(z) = H_0^2(z) - H_1^2(z) = H_0^2(z) - H_0^2(-z) = 4z^{-1}P_0(z^2)P_1(z^2)$

Options:

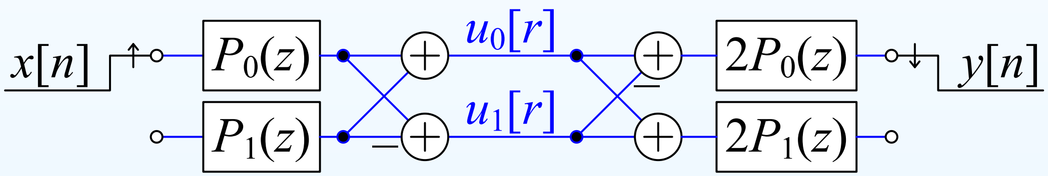
(A) Perfect Reconstruction: $T(z) = z^{-d} \Rightarrow H_0(z)$ is a bad filter.

(B) $T(z)$ is Linear Phase FIR:
$\Rightarrow$ Tradeoff: $\left|T(e^{j\omega})\right| \approx 1$ versus $H_0(z)$ stopband attenuation

QMF Options



Polyphase decomposition:



$A(z) = 0 \Rightarrow$ no alias term

$T(z) = H_0^2(z) - H_1^2(z) = H_0^2(z) - H_0^2(-z) = 4z^{-1}P_0(z^2)P_1(z^2)$

Options:

(A) Perfect Reconstruction: $T(z) = z^{-d} \Rightarrow H_0(z)$ is a bad filter.

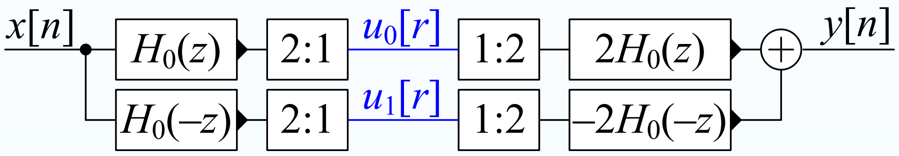
(B) $T(z)$ is Linear Phase FIR:

$\Rightarrow$ Tradeoff: $\left|T(e^{j\omega})\right| \approx 1$ versus $H_0(z)$ stopband attenuation

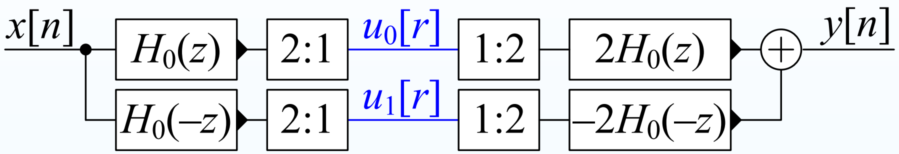
(C) $T(z)$ is Allpass IIR: $H_0(z)$ can be Butterworth or Elliptic filter

$\Rightarrow$ Tradeoff: $\angle T(e^{j\omega}) \approx \tau\omega$ versus $H_0(z)$ stopband attenuation

Option (B): Linear Phase QMF

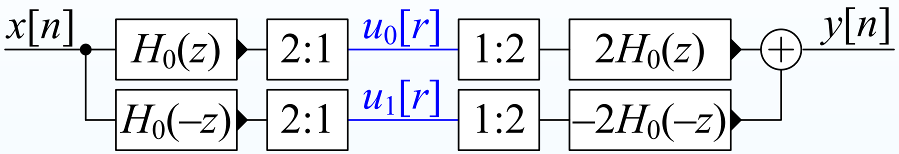


Option (B): Linear Phase QMF



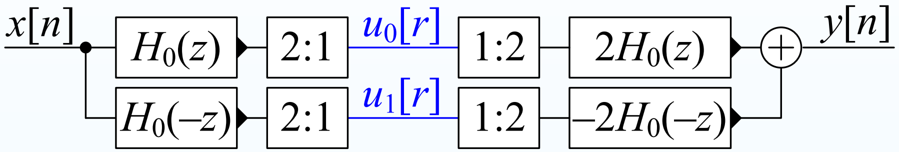
$H_0(z)$ order M, linear phase

Option (B): Linear Phase QMF



$H_0(z)$ order M, linear phase $\Rightarrow H_0(e^{j\omega}) = \pm e^{-j\omega\frac{M}{2}}\left|H_0(e^{j\omega})\right|$

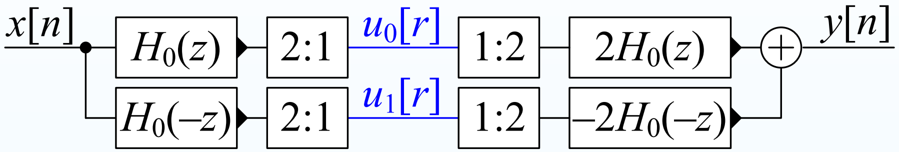
Option (B): Linear Phase QMF



$H_0(z)$ order M, linear phase $\Rightarrow H_0(e^{j\omega}) = \pm e^{-j\omega\frac{M}{2}} \left|H_0(e^{j\omega})\right|$

$T(e^{j\omega}) = H_0^2(e^{j\omega}) - H_1^2(e^{j\omega}) = H_0^2(e^{j\omega}) - H_0^2(-e^{j\omega})$

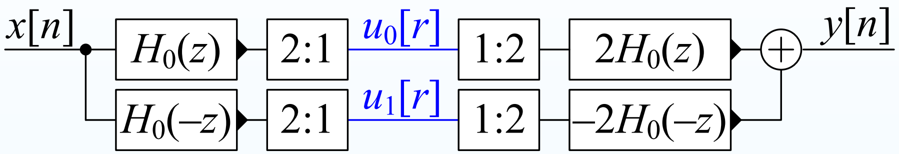
Option (B): Linear Phase QMF



$H_0(z)$ order M, linear phase $\Rightarrow H_0(e^{j\omega}) = \pm e^{-j\omega\frac{M}{2}} \left|H_0(e^{j\omega})\right|$

$$T(e^{j\omega}) = H_0^2(e^{j\omega}) - H_1^2(e^{j\omega}) = H_0^2(e^{j\omega}) - H_0^2(-e^{j\omega})$$
$$= e^{-j\omega M}\left|H_0(e^{j\omega})\right|^2 - e^{-j(\omega-\pi)M}\left|H_0(e^{j(\omega-\pi)})\right|^2$$

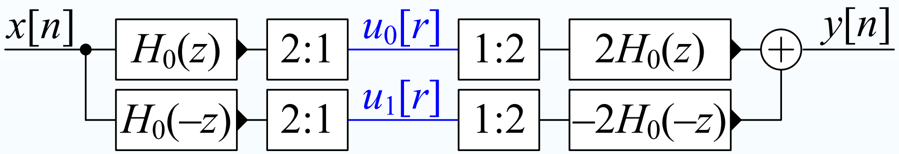
Option (B): Linear Phase QMF



$H_0(z)$ order M, linear phase $\Rightarrow H_0(e^{j\omega}) = \pm e^{-j\omega\frac{M}{2}} \left|H_0(e^{j\omega})\right|$

$$\begin{aligned}T(e^{j\omega}) &= H_0^2(e^{j\omega}) - H_1^2(e^{j\omega}) = H_0^2(e^{j\omega}) - H_0^2(-e^{j\omega}) \\ &= e^{-j\omega M} \left|H_0(e^{j\omega})\right|^2 - e^{-j(\omega-\pi)M} \left|H_0(e^{j(\omega-\pi)})\right|^2 \\ &= e^{-j\omega M} \left(\left|H_0(e^{j\omega})\right|^2 - (-1)^M \left|H_0(e^{j(\pi-\omega)})\right|^2\right)\end{aligned}$$

Option (B): Linear Phase QMF



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$$T(e^{j\omega}) = H_0^2(e^{j\omega}) - H_1^2(e^{j\omega}) = H_0^2(e^{j\omega}) - H_0^2(-e^{j\omega})$$
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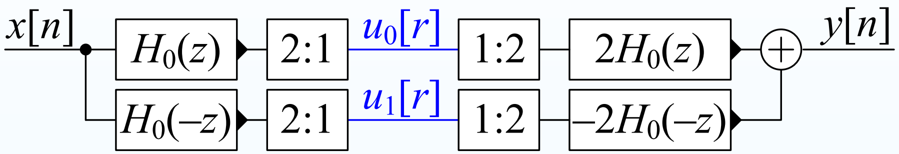
$$= e^{-j\omega M} \left(\left|H_0(e^{j\omega})\right|^2 - (-1)^M \left|H_0(e^{j(\pi-\omega)})\right|^2\right)$$

M even $\Rightarrow T(e^{j\frac{\pi}{2}}) = 0$ ☹

Option (B): Linear Phase QMF



$H_0(z)$ order M, linear phase $\Rightarrow H_0(e^{j\omega}) = \pm e^{-j\omega\frac{M}{2}}\left|H_0(e^{j\omega})\right|$

$$T(e^{j\omega}) = H_0^2(e^{j\omega}) - H_1^2(e^{j\omega}) = H_0^2(e^{j\omega}) - H_0^2(-e^{j\omega})$$
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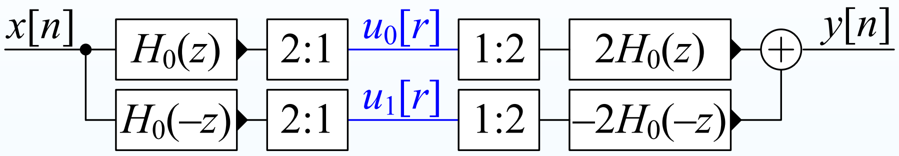
M even $\Rightarrow T(e^{j\frac{\pi}{2}}) = 0$ ☹ so choose M odd $\Rightarrow -(-1)^M = +1$

Option (B): Linear Phase QMF



$H_0(z)$ order M, linear phase $\Rightarrow H_0(e^{j\omega}) = \pm e^{-j\omega\frac{M}{2}}\left|H_0(e^{j\omega})\right|$

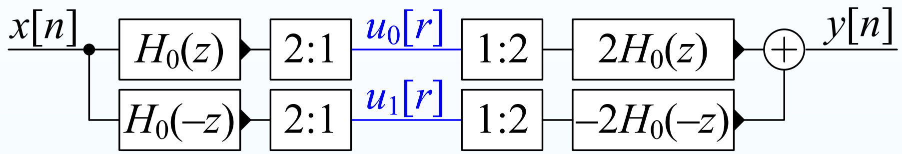
$$\begin{aligned} T(e^{j\omega}) &= H_0^2(e^{j\omega}) - H_1^2(e^{j\omega}) = H_0^2(e^{j\omega}) - H_0^2(-e^{j\omega}) \\ &= e^{-j\omega M}\left|H_0(e^{j\omega})\right|^2 - e^{-j(\omega-\pi)M}\left|H_0(e^{j(\omega-\pi)})\right|^2 \\ &= e^{-j\omega M}\left(\left|H_0(e^{j\omega})\right|^2 - (-1)^M\left|H_0(e^{j(\pi-\omega)})\right|^2\right) \end{aligned}$$

M even $\Rightarrow T(e^{j\frac{\pi}{2}}) = 0$ ☹ so choose M odd $\Rightarrow -(-1)^M = +1$

Select $h_0[n]$ by numerical iteration to minimize

$$\alpha\int_{\frac{\pi}{2}+\Delta}^{\pi}\left|H_0(e^{j\omega})\right|^2 d\omega + (1-\alpha)\int_0^{\pi}\left(\left|T(e^{j\omega})\right| - 1\right)^2 d\omega$$

Option (B): Linear Phase QMF



$H_0(z)$ order M, linear phase $\Rightarrow H_0(e^{j\omega}) = \pm e^{-j\omega\frac{M}{2}} \left|H_0(e^{j\omega})\right|$

$$\begin{aligned} T(e^{j\omega}) &= H_0^2(e^{j\omega}) - H_1^2(e^{j\omega}) = H_0^2(e^{j\omega}) - H_0^2(-e^{j\omega}) \\ &= e^{-j\omega M} \left|H_0(e^{j\omega})\right|^2 - e^{-j(\omega-\pi)M} \left|H_0(e^{j(\omega-\pi)})\right|^2 \\ &= e^{-j\omega M} \left(\left|H_0(e^{j\omega})\right|^2 - (-1)^M \left|H_0(e^{j(\pi-\omega)})\right|^2 \right) \end{aligned}$$

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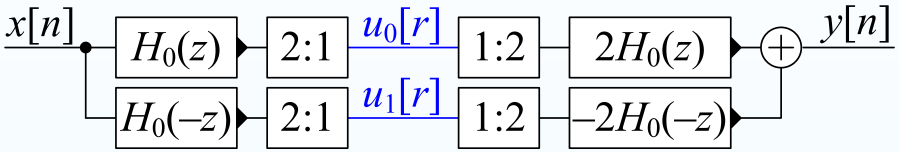
Select $h_0[n]$ by numerical iteration to minimize

$$\alpha \int_{\frac{\pi}{2}+\Delta}^{\pi} \left|H_0(e^{j\omega})\right|^2 d\omega + (1-\alpha) \int_0^{\pi} \left(\left|T(e^{j\omega})\right| - 1 \right)^2 d\omega$$

$\alpha \rightarrow$ balance between $H_0(z)$ being lowpass and $T(e^{j\omega}) \approx 1$

Option (B): Linear Phase QMF



$H_0(z)$ order M, linear phase $\Rightarrow H_0(e^{j\omega}) = \pm e^{-j\omega\frac{M}{2}}\left|H_0(e^{j\omega})\right|$

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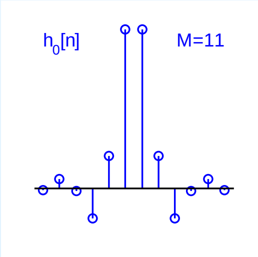
M even $\Rightarrow T(e^{j\frac{\pi}{2}}) = 0$ ☹ so choose M odd $\Rightarrow -(-1)^M = +1$

Select $h_0[n]$ by numerical iteration to minimize

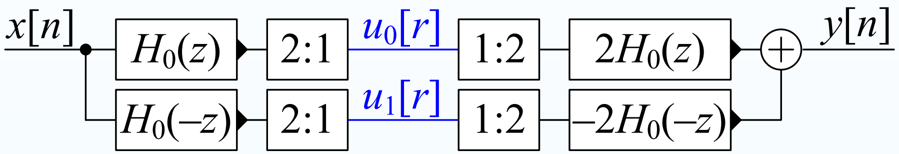
$$\alpha\int_{\frac{\pi}{2}+\Delta}^{\pi}\left|H_0(e^{j\omega})\right|^2 d\omega + (1-\alpha)\int_0^{\pi}\left(\left|T(e^{j\omega})\right|-1\right)^2 d\omega$$

$\alpha \rightarrow$ balance between $H_0(z)$ being lowpass and $T(e^{j\omega}) \approx 1$

Johnston filter ($M = 11$):



Option (B): Linear Phase QMF



$H_0(z)$ order M, linear phase $\Rightarrow H_0(e^{j\omega}) = \pm e^{-j\omega\frac{M}{2}} \left|H_0(e^{j\omega})\right|$

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M even $\Rightarrow T(e^{j\frac{\pi}{2}}) = 0$ ☹ so choose M odd $\Rightarrow -(-1)^M = +1$

Select $h_0[n]$ by numerical iteration to minimize

$$\alpha \int_{\frac{\pi}{2}+\Delta}^{\pi} \left|H_0(e^{j\omega})\right|^2 d\omega + (1-\alpha)\int_0^{\pi}\left(\left|T(e^{j\omega})\right| - 1\right)^2 d\omega$$

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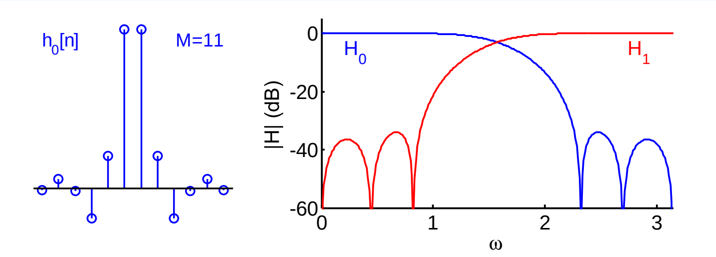
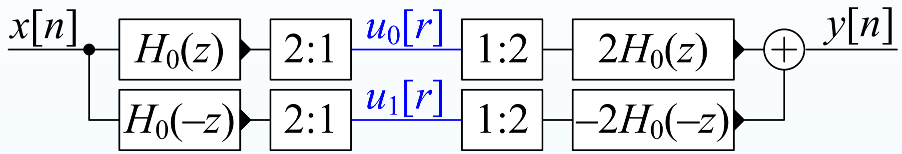
Johnston filter ($M = 11$):



Option (B): Linear Phase QMF



$H_0(z)$ order M, linear phase $\Rightarrow H_0(e^{j\omega}) = \pm e^{-j\omega\frac{M}{2}}\left|H_0(e^{j\omega})\right|$

$$T(e^{j\omega}) = H_0^2(e^{j\omega}) - H_1^2(e^{j\omega}) = H_0^2(e^{j\omega}) - H_0^2(-e^{j\omega})$$
$$= e^{-j\omega M}\left|H_0(e^{j\omega})\right|^2 - e^{-j(\omega-\pi)M}\left|H_0(e^{j(\omega-\pi)})\right|^2$$
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M even $\Rightarrow T(e^{j\frac{\pi}{2}}) = 0$ ☹ so choose M odd $\Rightarrow -(-1)^M = +1$

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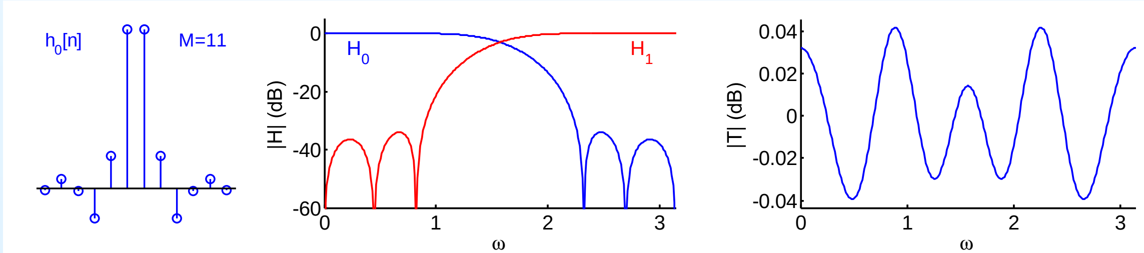
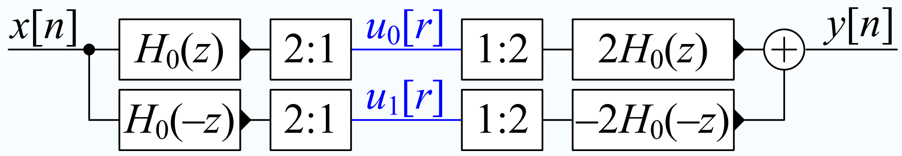
Johnston filter ($M = 11$):



Option (B): Linear Phase QMF



$T(z) \approx 1$

$H_0(z)$ order M, linear phase $\Rightarrow H_0(e^{j\omega}) = \pm e^{-j\omega\frac{M}{2}} \left|H_0(e^{j\omega})\right|$

$$T(e^{j\omega}) = H_0^2(e^{j\omega}) - H_1^2(e^{j\omega}) = H_0^2(e^{j\omega}) - H_0^2(-e^{j\omega})$$
$$= e^{-j\omega M} \left|H_0(e^{j\omega})\right|^2 - e^{-j(\omega-\pi)M} \left|H_0(e^{j(\omega-\pi)})\right|^2$$
$$= e^{-j\omega M} \left(\left|H_0(e^{j\omega})\right|^2 - (-1)^M \left|H_0(e^{j(\pi-\omega)})\right|^2\right)$$

M even $\Rightarrow T(e^{j\frac{\pi}{2}}) = 0$ ☹ so choose M odd $\Rightarrow -(-1)^M = +1$

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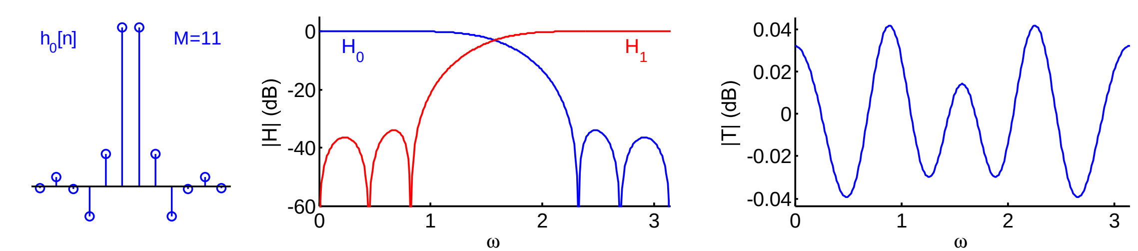
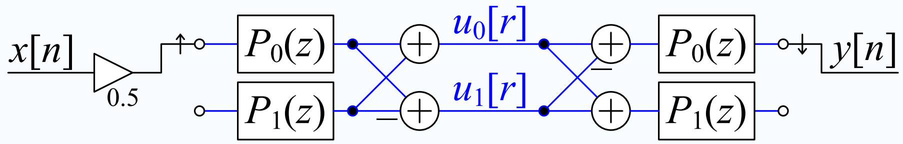
$$\alpha \int_{\frac{\pi}{2}+\Delta}^{\pi} \left|H_0(e^{j\omega})\right|^2 d\omega + (1-\alpha) \int_0^{\pi} \left(\left|T(e^{j\omega})\right| - 1\right)^2 d\omega$$

$\alpha \to$ balance between $H_0(z)$ being lowpass and $T(e^{j\omega}) \approx 1$

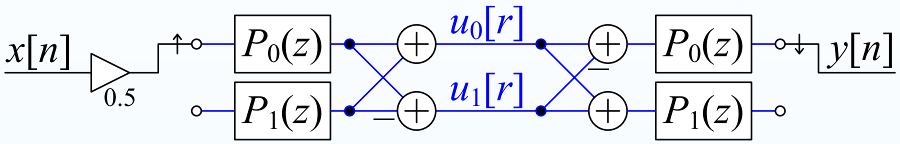
Johnston filter ($M = 11$):



Option (C): IIR Allpass QMF

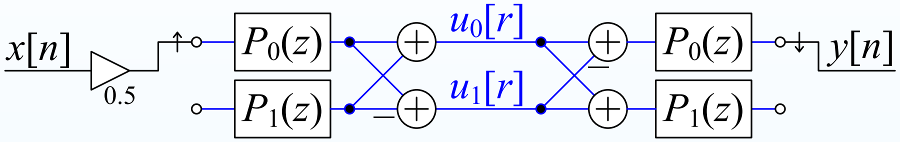


Option (C): IIR Allpass QMF



Choose $P_0(z)$ and $P_1(z)$ to be allpass IIR filters:

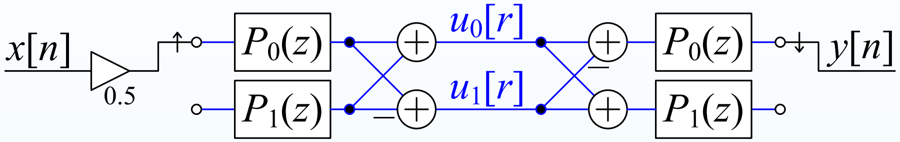
Option (C): IIR Allpass QMF



Choose $P_0(z)$ and $P_1(z)$ to be allpass IIR filters:

$$H_{0,1}(z) = \frac{1}{2}\left(P_0(z^2) \pm z^{-1}P_1(z^2)\right), \qquad G_{0,1}(z) = \pm 2H_{0,1}(z)$$

Option (C): IIR Allpass QMF



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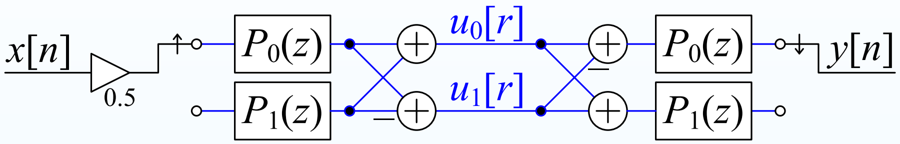
$A(z) = 0 \Rightarrow$ No aliasing

Option (C): IIR Allpass QMF



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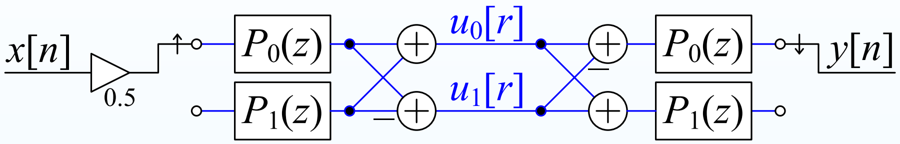
$A(z) = 0 \Rightarrow$ No aliasing

$T(z) = H_0^2 - H_1^2 = \ldots = z^{-1}P_0(z^2)P_1(z^2)$

Option (C): IIR Allpass QMF



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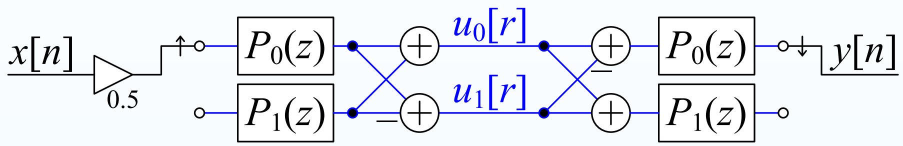
$A(z) = 0 \Rightarrow$ No aliasing

$T(z) = H_0^2 - H_1^2 = \ldots = z^{-1}P_0(z^2)P_1(z^2)$ is an allpass filter.

Option (C): IIR Allpass QMF



Choose $P_0(z)$ and $P_1(z)$ to be allpass IIR filters:

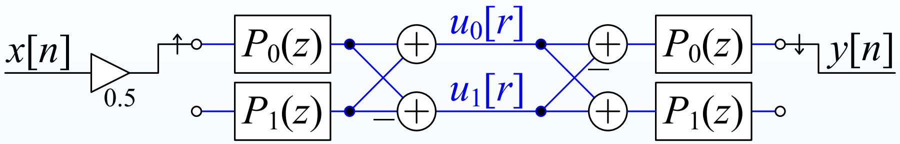
$$H_{0,1}(z) = \frac{1}{2}\left(P_0(z^2) \pm z^{-1}P_1(z^2)\right), \qquad G_{0,1}(z) = \pm 2H_{0,1}(z)$$

$A(z) = 0 \Rightarrow$ No aliasing

$T(z) = H_0^2 - H_1^2 = \ldots = z^{-1}P_0(z^2)P_1(z^2)$ is an allpass filter.

$H_0(z)$ can be made a Butterworth or Elliptic filter with $M_H = 4M_P + 1$:

Option (C): IIR Allpass QMF



Choose $P_0(z)$ and $P_1(z)$ to be allpass IIR filters:

$$H_{0,1}(z) = \frac{1}{2}\left(P_0(z^2) \pm z^{-1}P_1(z^2)\right), \qquad G_{0,1}(z) = \pm 2H_{0,1}(z)$$

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$T(z) = H_0^2 - H_1^2 = \ldots = z^{-1}P_0(z^2)P_1(z^2)$ is an allpass filter.

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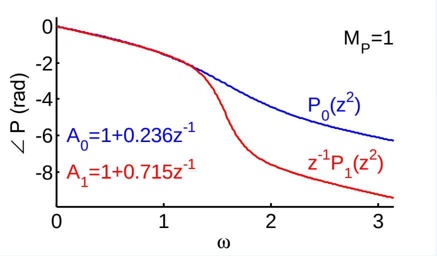

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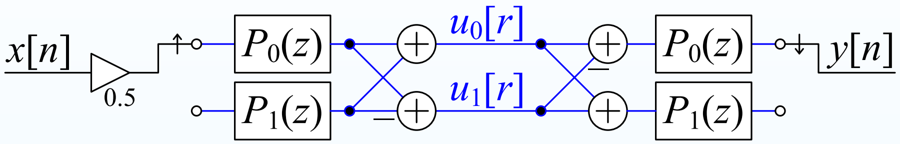
Option (C): IIR Allpass QMF



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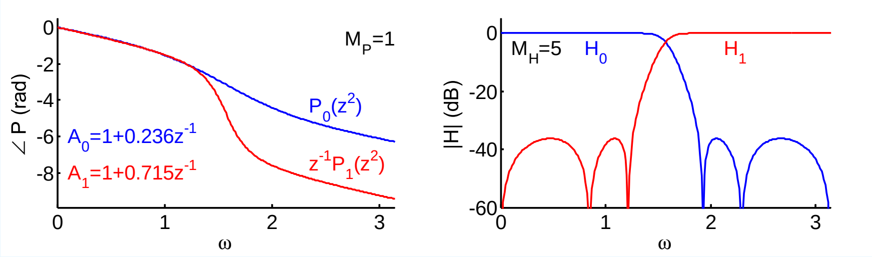

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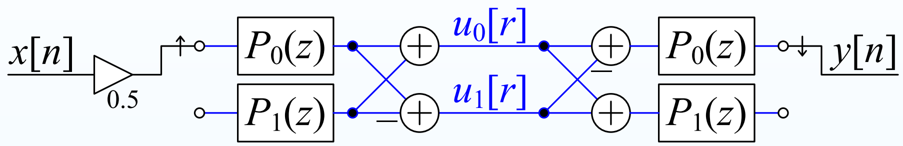

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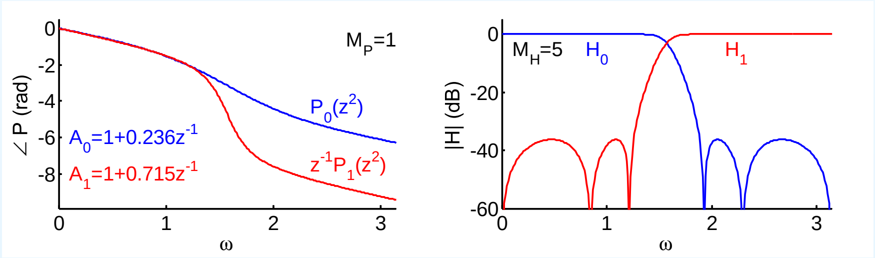

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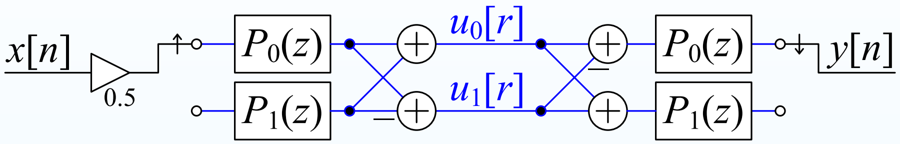

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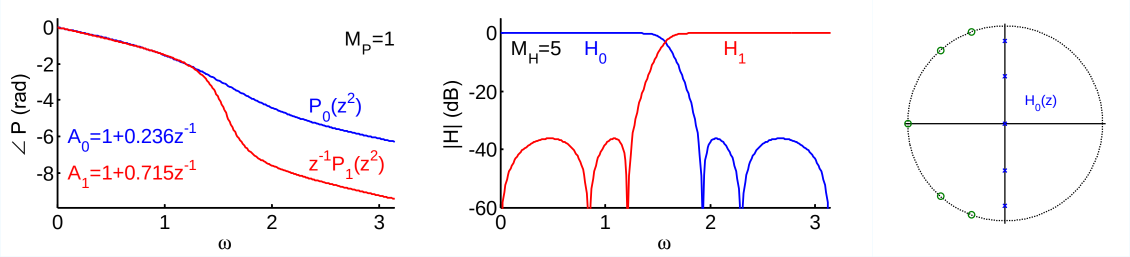

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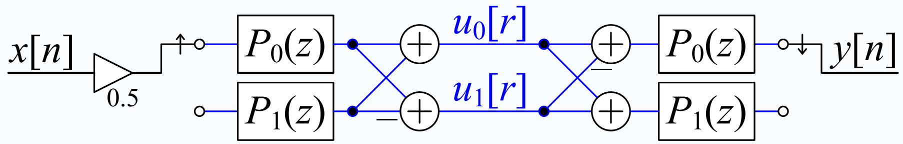

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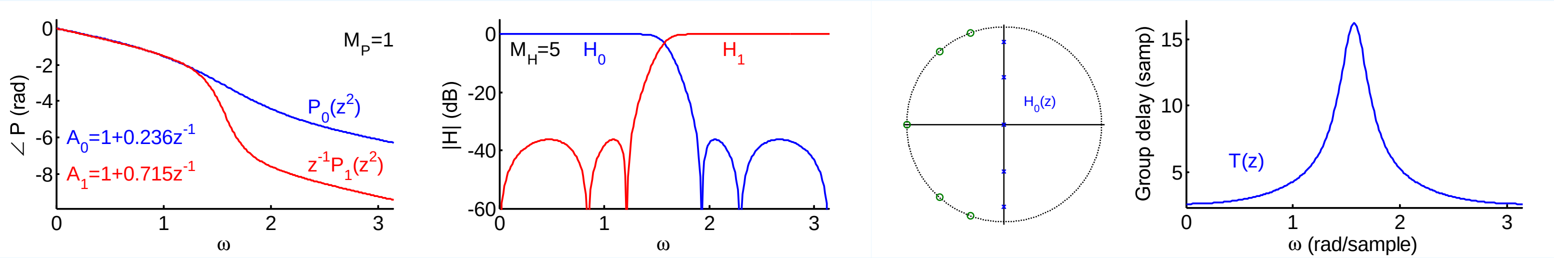

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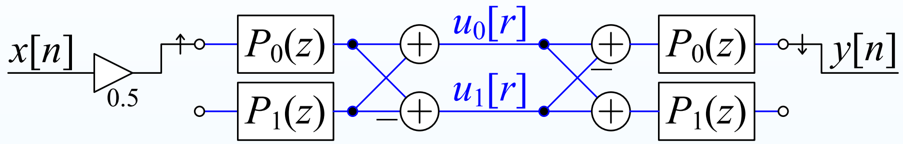

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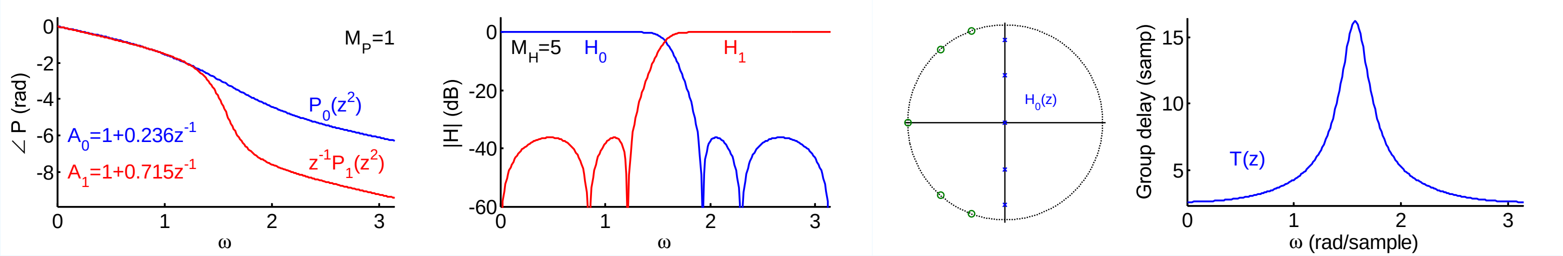

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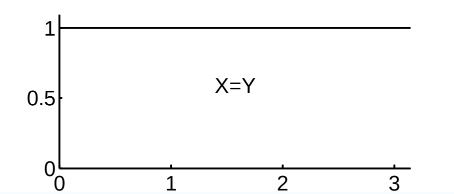
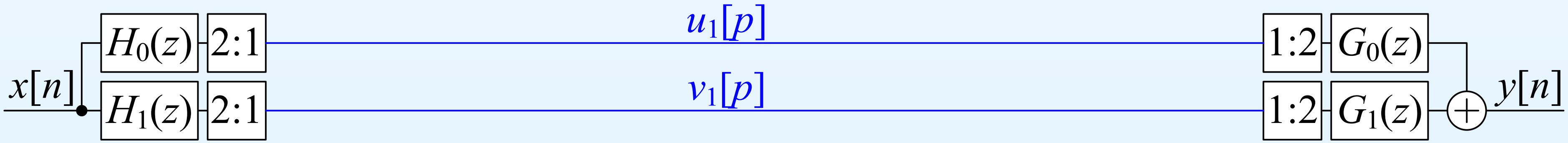
Tree-structured filterbanks

A *half-band filterbank* divides the full band into two equal halves.





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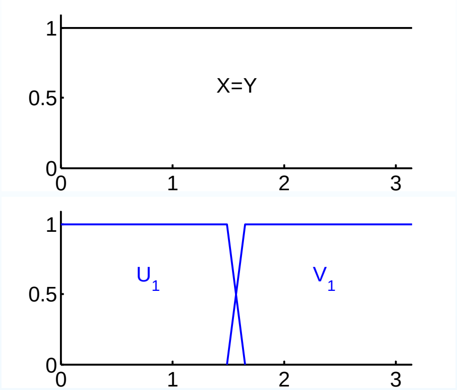

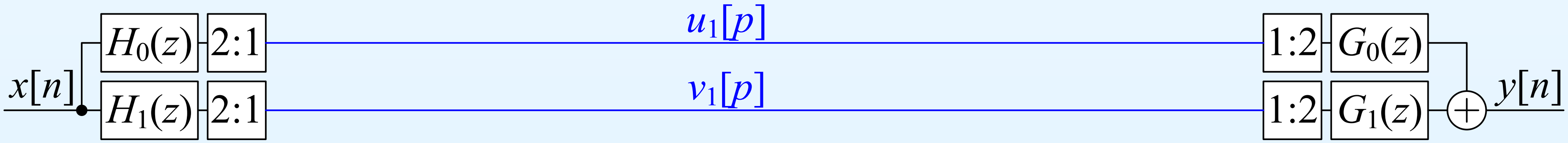

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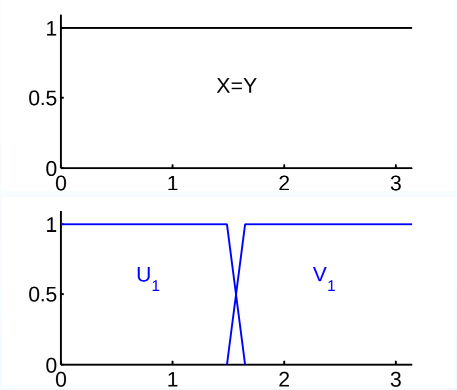

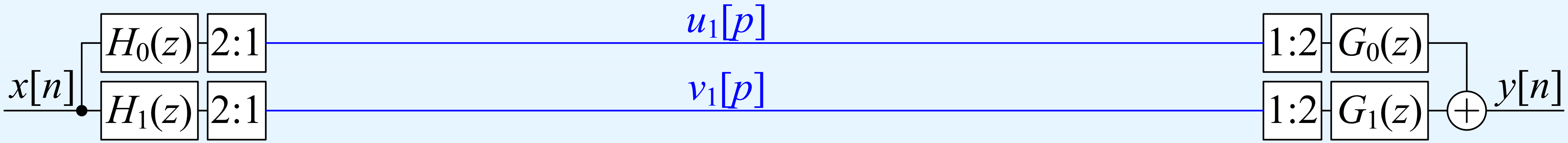

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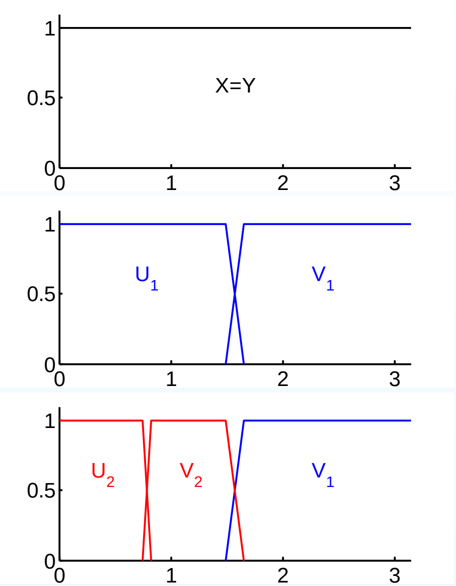

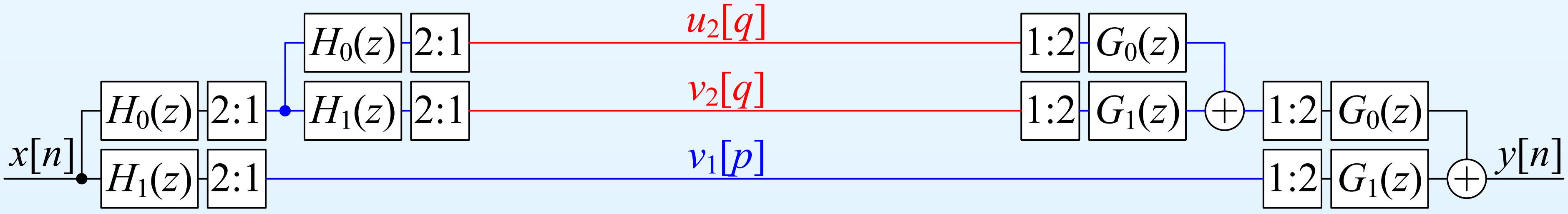

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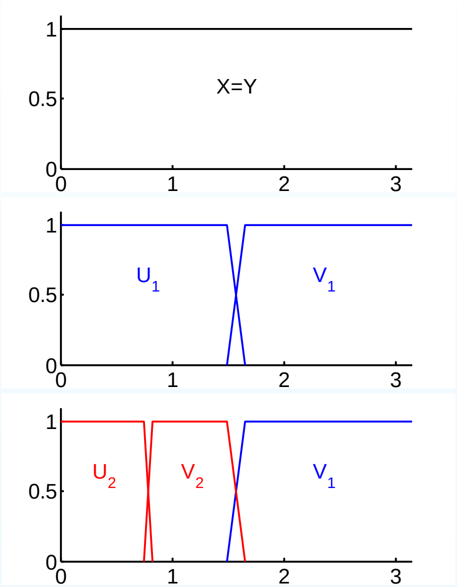

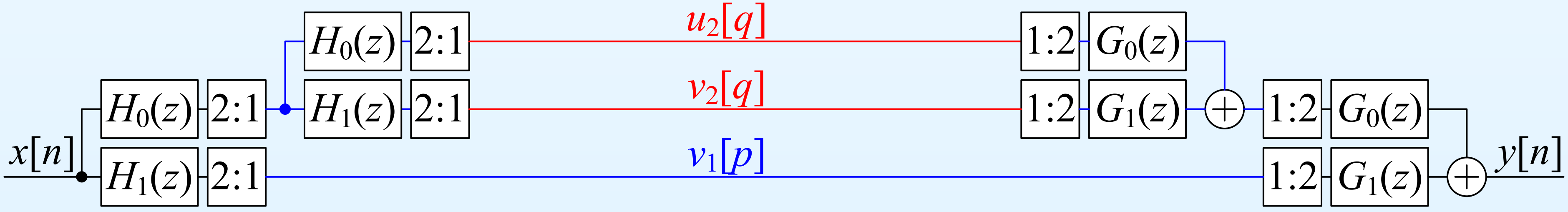

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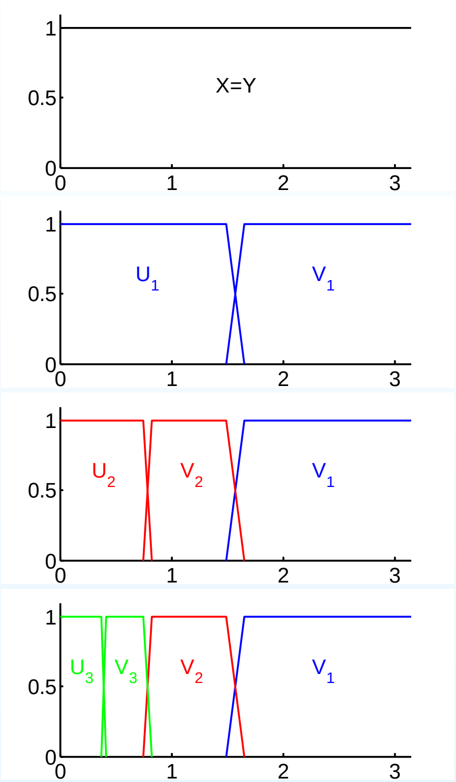

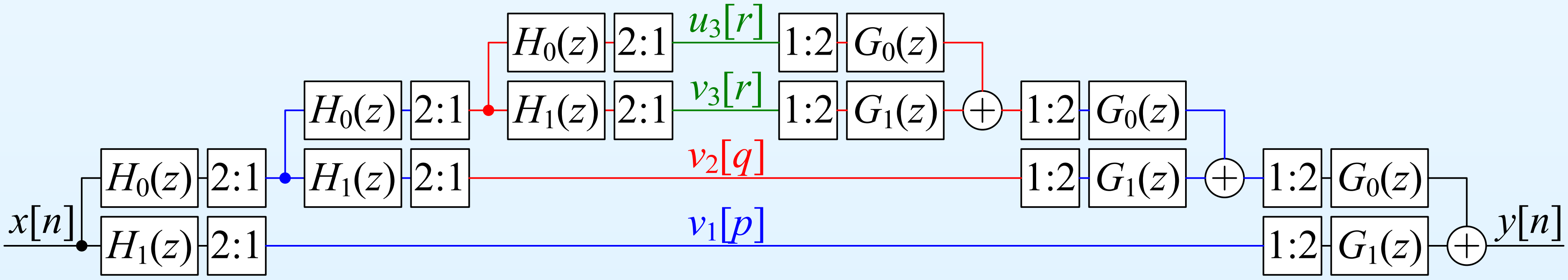

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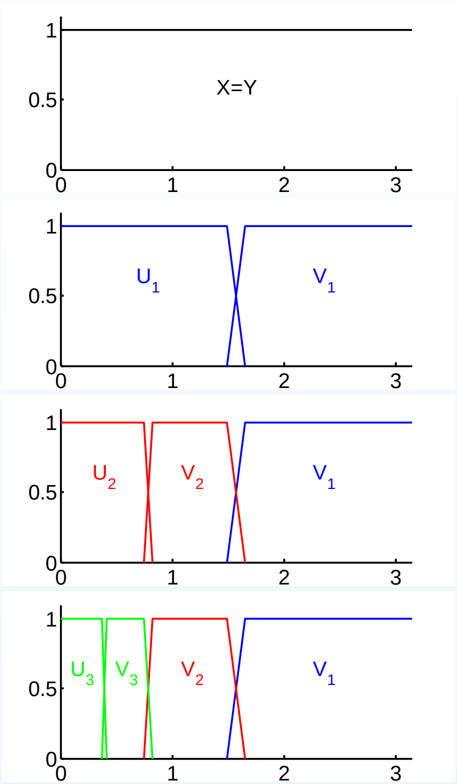

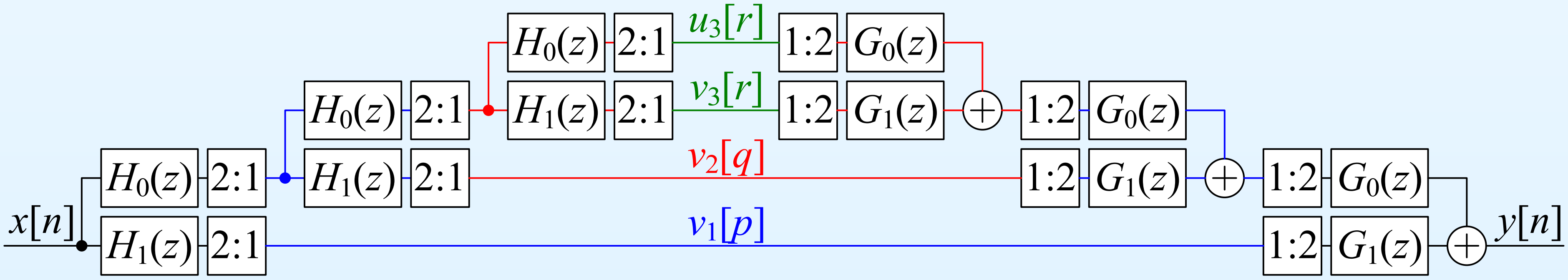

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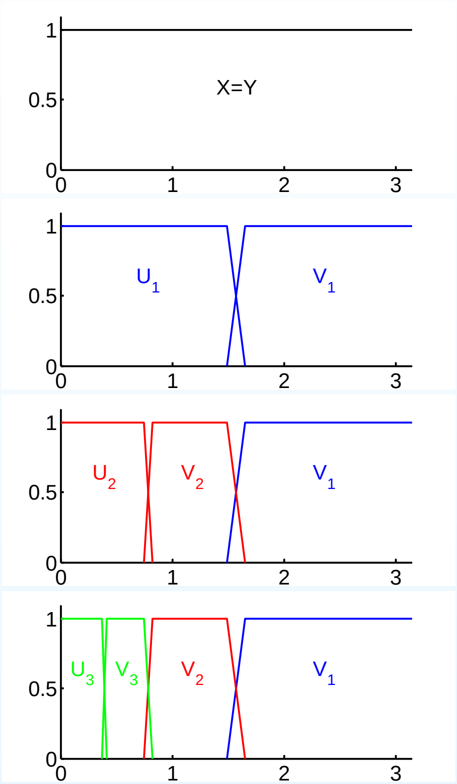
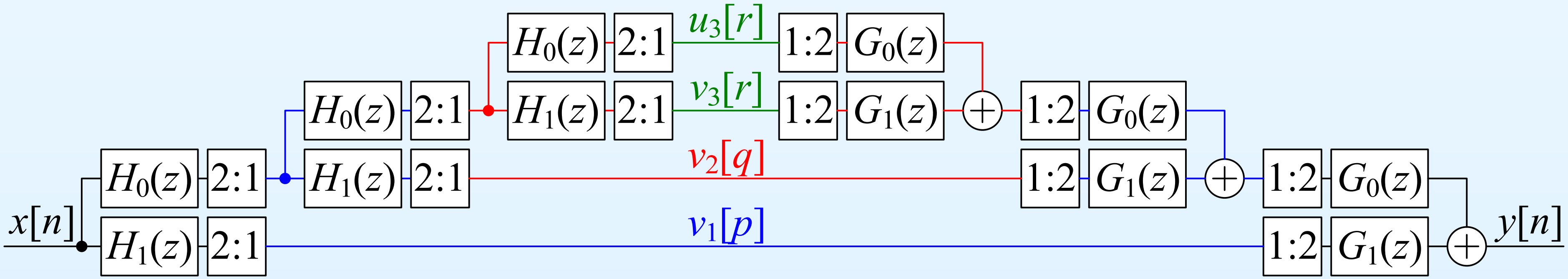
The properties "perfect reconstruction" and "allpass" are preserved by the iteration.





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  - Perfect reconstruction now impossible except for trivial case.
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See Mitra chapter 14 (which also includes some perfect reconstruction designs).

Merry Xmas