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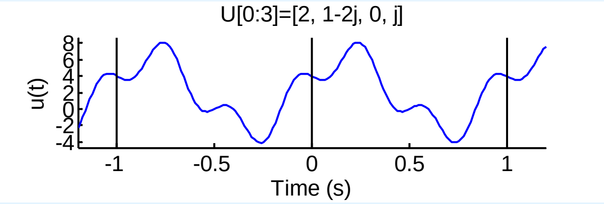

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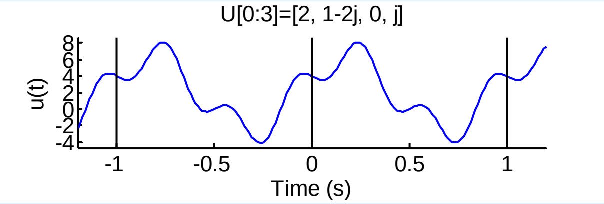

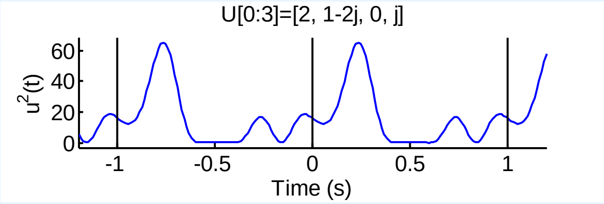

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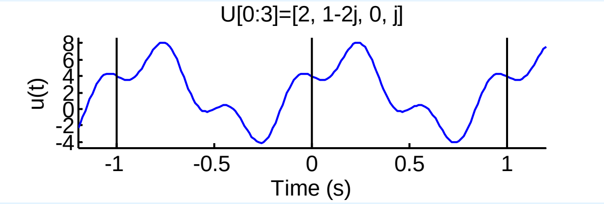

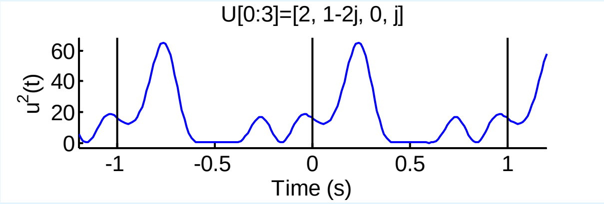

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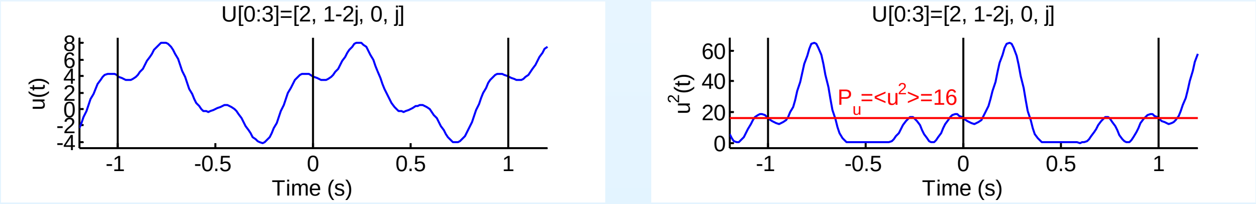

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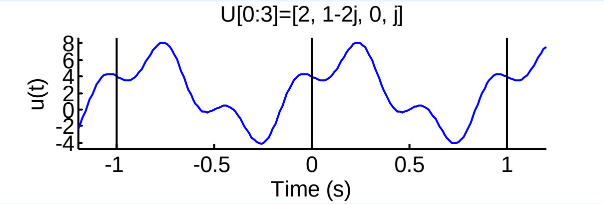
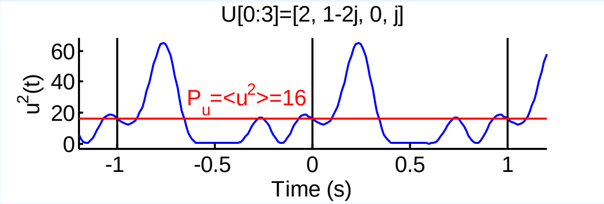
$= \frac{1}{4}a_0^2 + \frac{1}{2}\sum_{n=1}^{\infty}\left(a_n^2 + b_n^2\right)$ [$U_{+n} = \frac{a_n - ib_n}{2}$]

Parseval's theorem $\Rightarrow$ the average power in $u(t)$ is equal to the sum of the average powers in each of its Fourier components.

Example: $u(t) = 2 + 2\cos 2\pi Ft + 4\sin 2\pi Ft - 2\sin 6\pi Ft$

$$\left\langle |u(t)|^2 \right\rangle = 4 + \frac{1}{2}\left(2^2 + 4^2 + (-2)^2\right) = 16$$





$U_{0:3} = [2,\ 1-2i,\ 0,\ i]$

Power Conservation

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- Convolution Example
- Convolution and Polynomial Multiplication
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The average power of a periodic signal is given by $P_u \triangleq \left\langle |u(t)|^2 \right\rangle$.

This is the average electrical power that would be dissipated if the signal represents the voltage across a $1\,\Omega$ resistor.

Parseval's Theorem: $P_u = \left\langle |u(t)|^2 \right\rangle = \sum_{n=-\infty}^{\infty} |U_n|^2$

$$= |U_0|^2 + 2\sum_{n=1}^{\infty} |U_n|^2 \qquad \text{[assume } u(t) \text{ real]}$$

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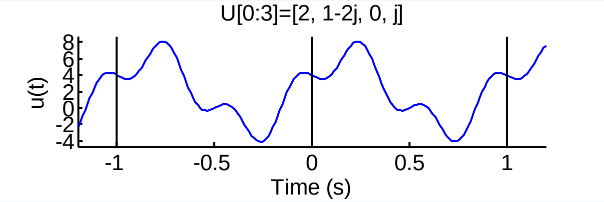

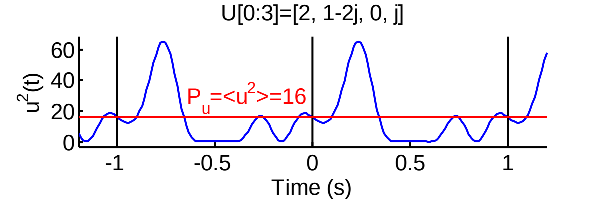

$$U_{0:3} = [2,\ 1-2i,\ 0,\ i] \quad \Rightarrow \quad |U_0|^2 + 2\sum_{n=1}^{\infty}|U_n|^2 = 16$$

Magnitude Spectrum and Power Spectrum

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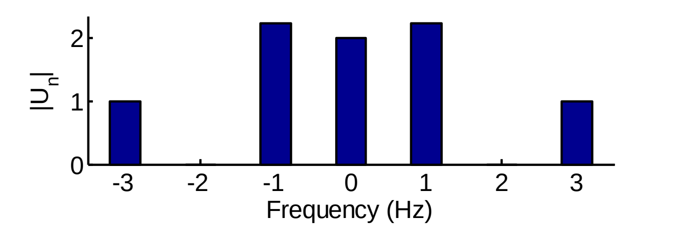

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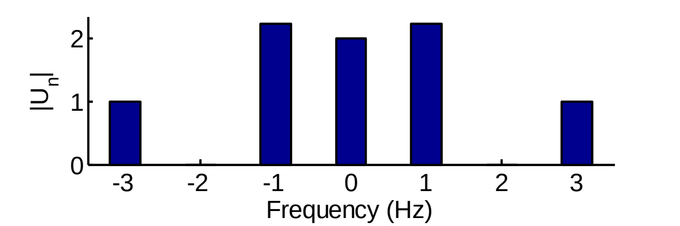

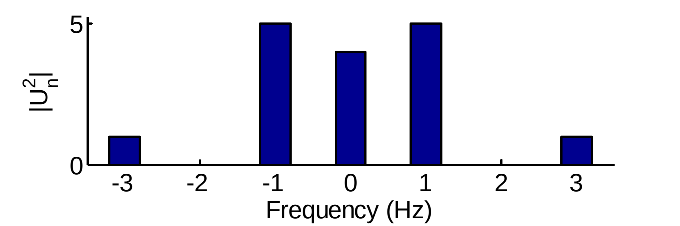

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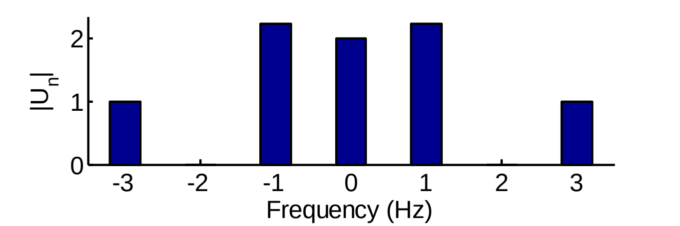

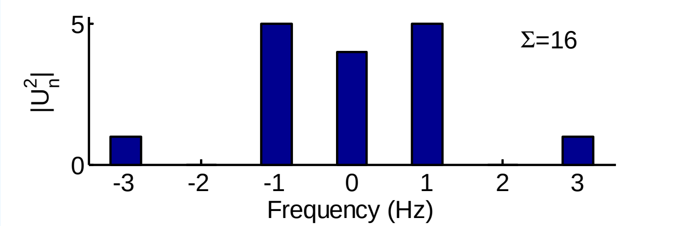

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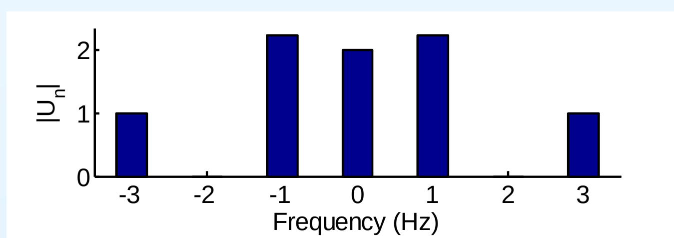

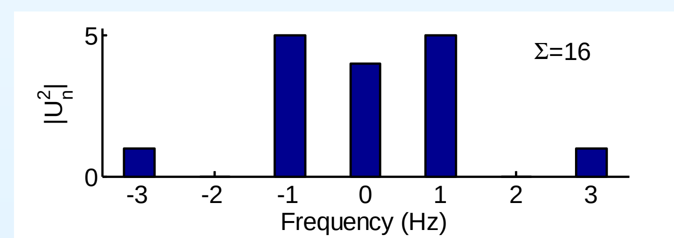

The magnitude and power spectra of a real periodic signal are symmetrical.

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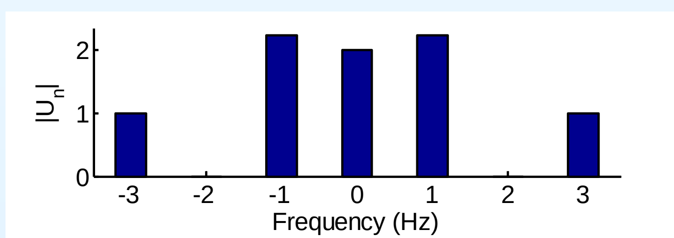

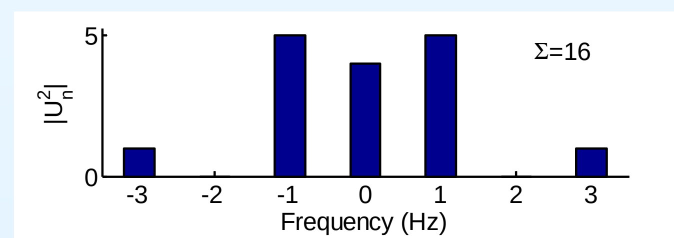

The magnitude and power spectra of a real periodic signal are symmetrical.

A *one-sided power power spectrum* shows U_0 and then $2\left|U_n\right|^2$ for $n \geq 1$.

Product of Signals

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Suppose we have two signals with the same period, $T = \frac{1}{F}$,

$$u(t) = \sum_{n=-\infty}^{\infty} U_n e^{i2\pi nFt}$$
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$$r = m + n \Rightarrow n = r - m$$

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This is a one-to-one mapping: every pair (m, n) in the range $\pm\infty$ corresponds to exactly one pair (m, r) in the same range.

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where $W_r = \sum_{m=-\infty}^{\infty} U_{r-m} V_m \triangleq U_r * V_r$.

W_r is the sum of all products $U_n V_m$ for which $m + n = r$.

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The spectrum $W_r = U_r * V_r$ is called the convolution of U_r and V_r.

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4) Identity Element or "1": If $I_r = \begin{cases} 1 & r = 0 \\ 0 & r \neq 0 \end{cases}$, then $I_r * U_r = U_r$

Proofs: (all sums are over $\pm\infty$)

1) Substitute for m: $n = r - m \Leftrightarrow m = r - n$ [$1 \leftrightarrow 1$ for any r]

$\sum_m U_{r-m} V_m = \sum_n U_n V_{r-n}$

2) Substitute for n: $k = r + m - n \Leftrightarrow n = r + m - k$ [$1 \leftrightarrow 1$]

$\sum_n \left(\left(\sum_m U_{n-m} V_m \right) W_{r-n} \right) = \sum_k \left(\left(\sum_m U_{r-k} V_m \right) W_{k-m} \right)$

$= \sum_k \sum_m U_{r-k} V_m W_{k-m} = \sum_k \left(U_{r-k} \left(\sum_m V_m W_{k-m} \right) \right)$

3) $\sum_m W_{r-m} (U_m + V_m) = \sum_m W_{r-m} U_m + \sum_m W_{r-m} V_m$

4) $I_{r-m} U_m = 0$ unless $m = r$. Hence $\sum_m I_{r-m} U_m = U_r$.

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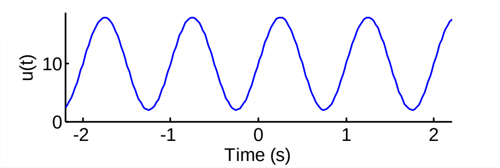
- Convolution and Polynomial Multiplication
- Summary

Convolution Example

$$u(t) = 10 + 8\sin 2\pi t$$



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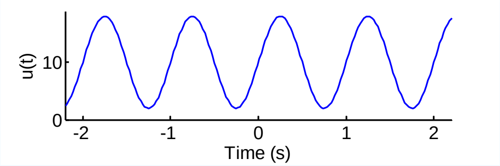

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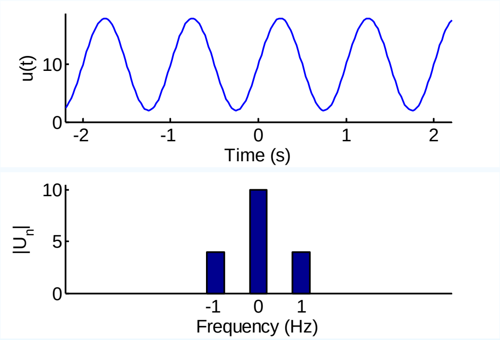

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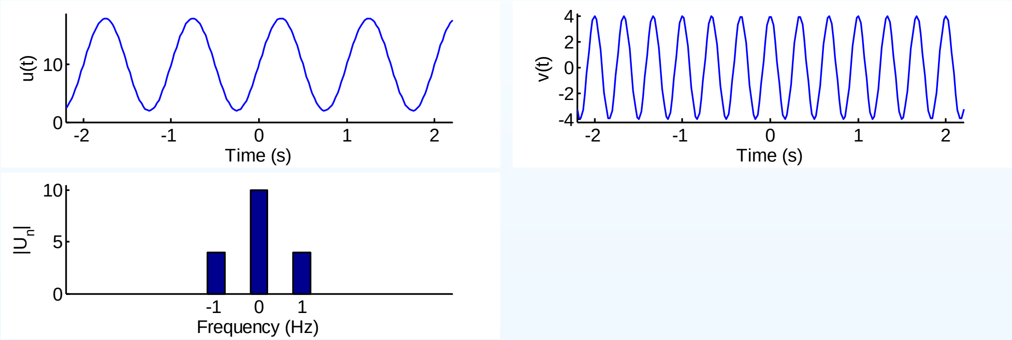

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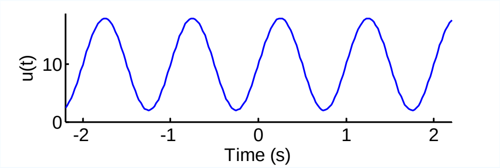

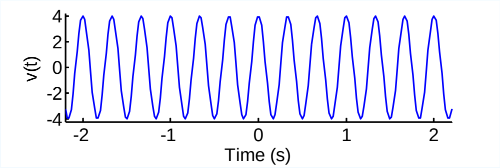

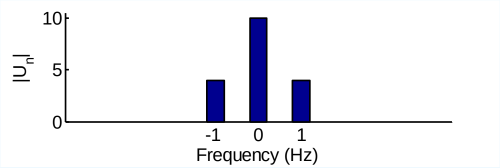

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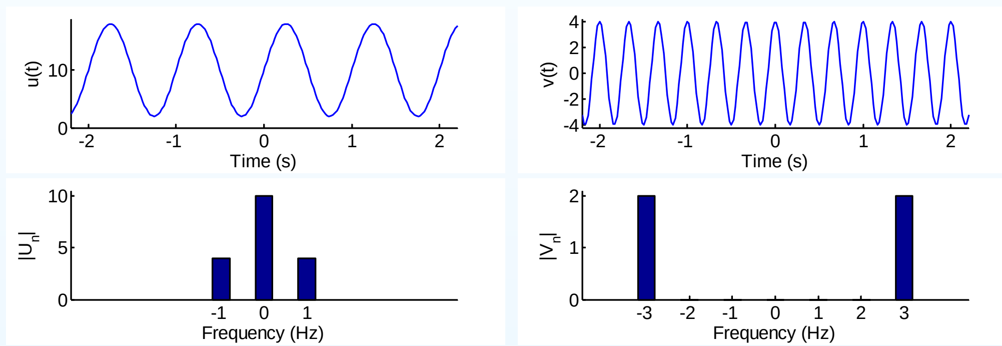

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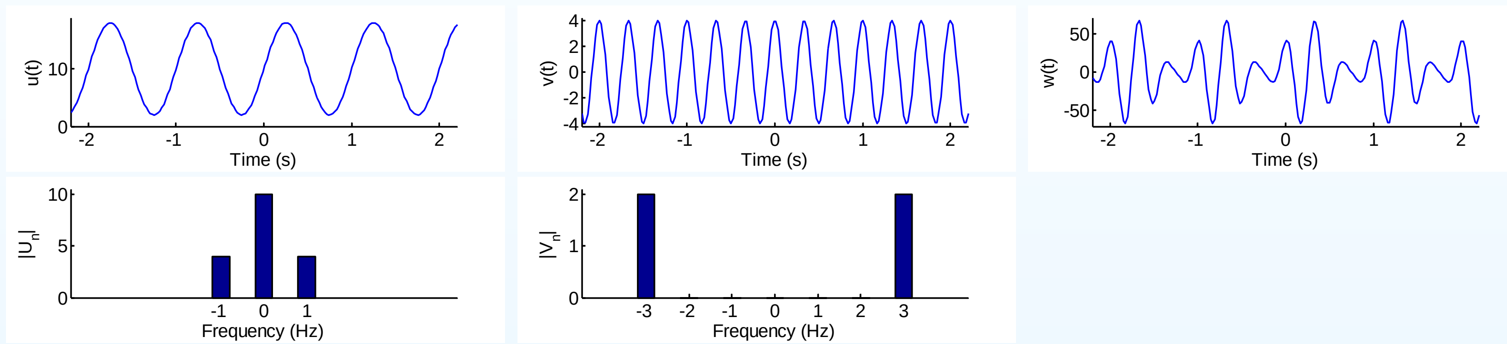

$$w(t) = u(t)v(t)$$

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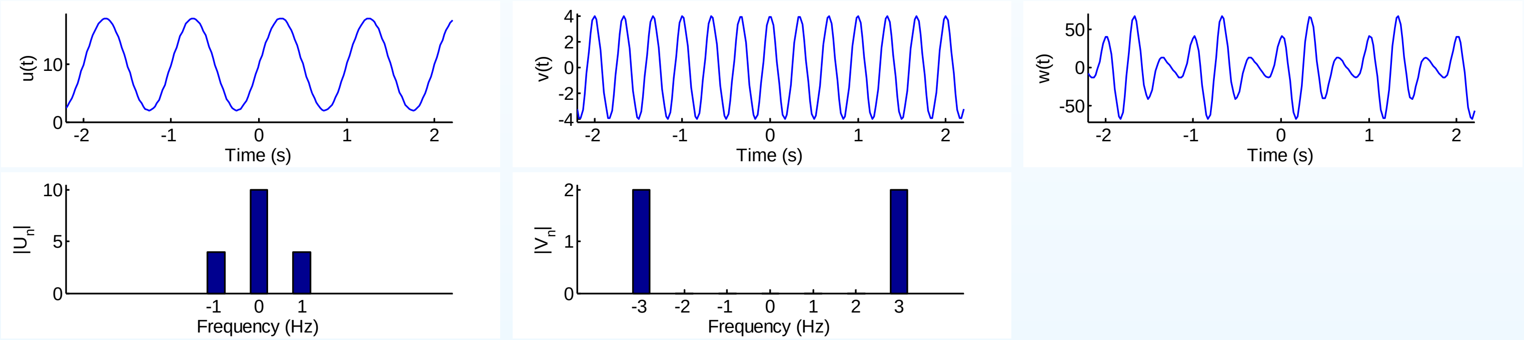

$$w(t) = u(t)v(t) = (10 + 8\sin 2\pi t)\, 4\cos 6\pi t$$

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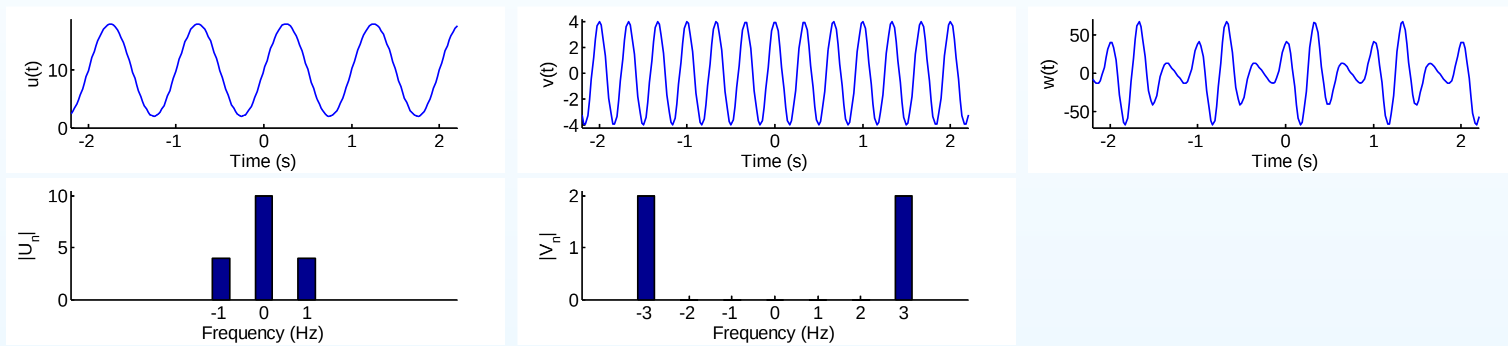

$$w(t) = u(t)v(t) = (10 + 8\sin 2\pi t)\, 4\cos 6\pi t$$
$$= 40\cos 6\pi t + 32\sin 2\pi t \cos 6\pi t$$

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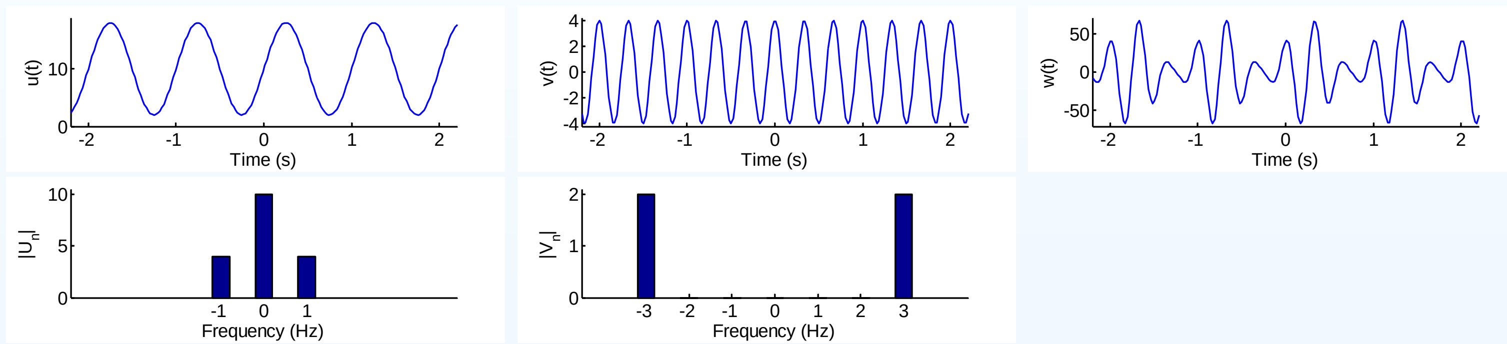

$$\begin{aligned} w(t) = u(t)v(t) &= (10 + 8\sin 2\pi t)\, 4\cos 6\pi t \\ &= 40\cos 6\pi t + 32\sin 2\pi t\cos 6\pi t \\ &= 40\cos 6\pi t + 16\sin 8\pi t - 16\sin 4\pi t \end{aligned}$$

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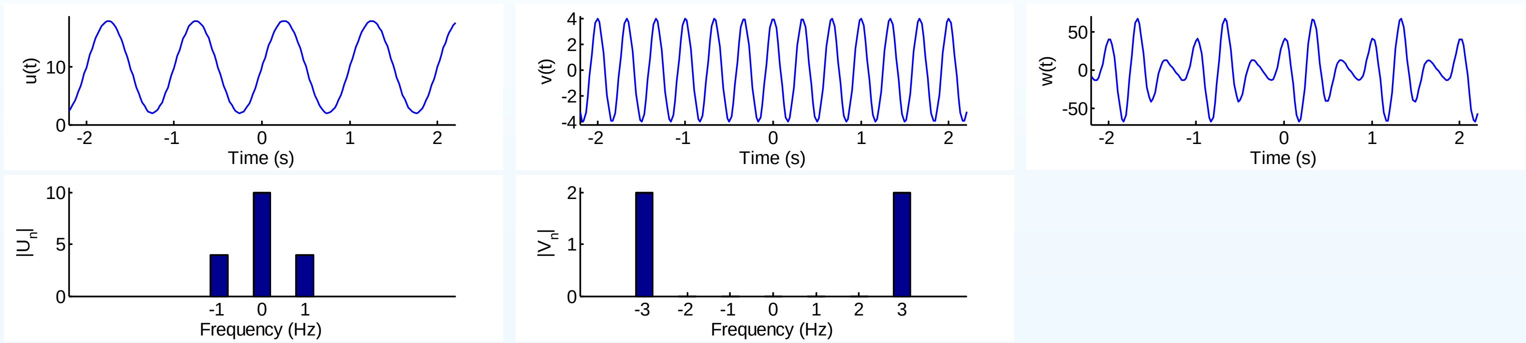

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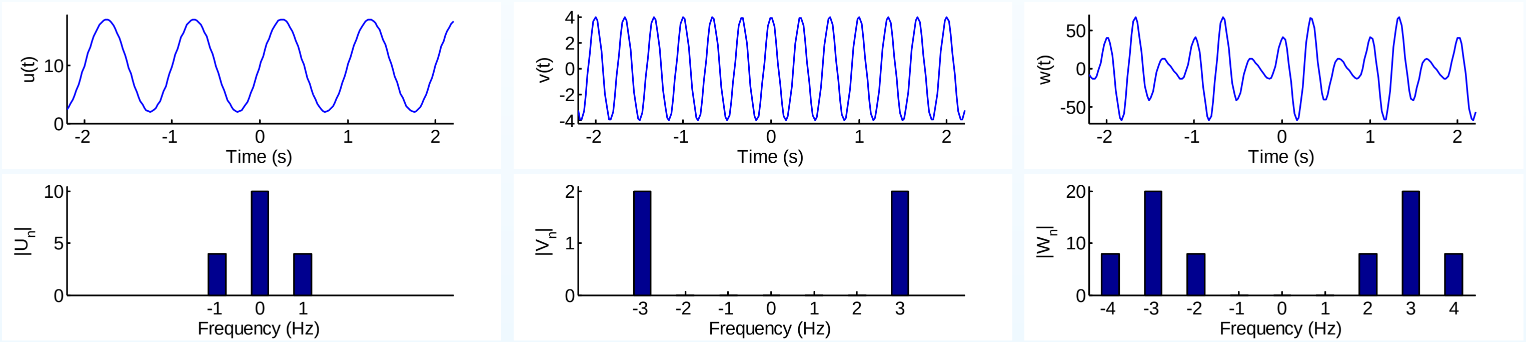

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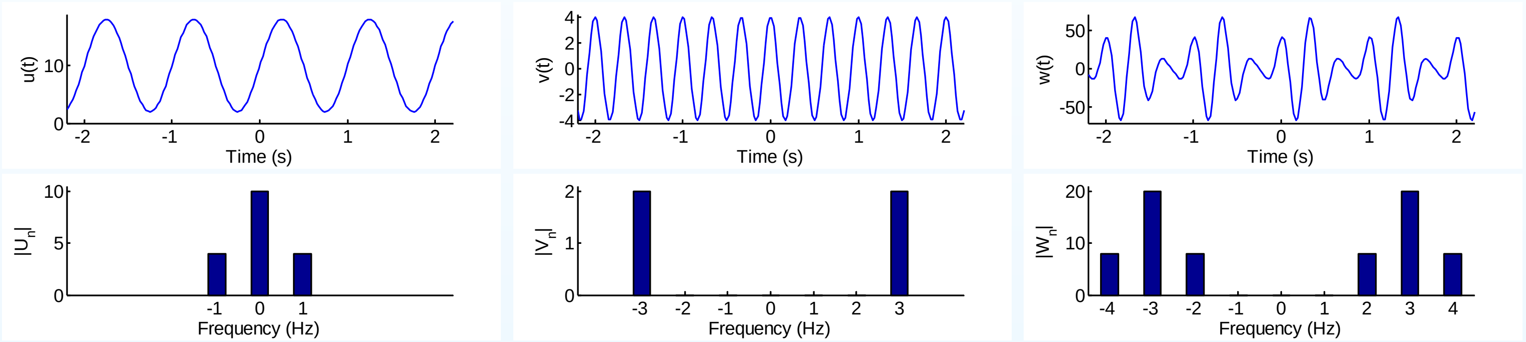

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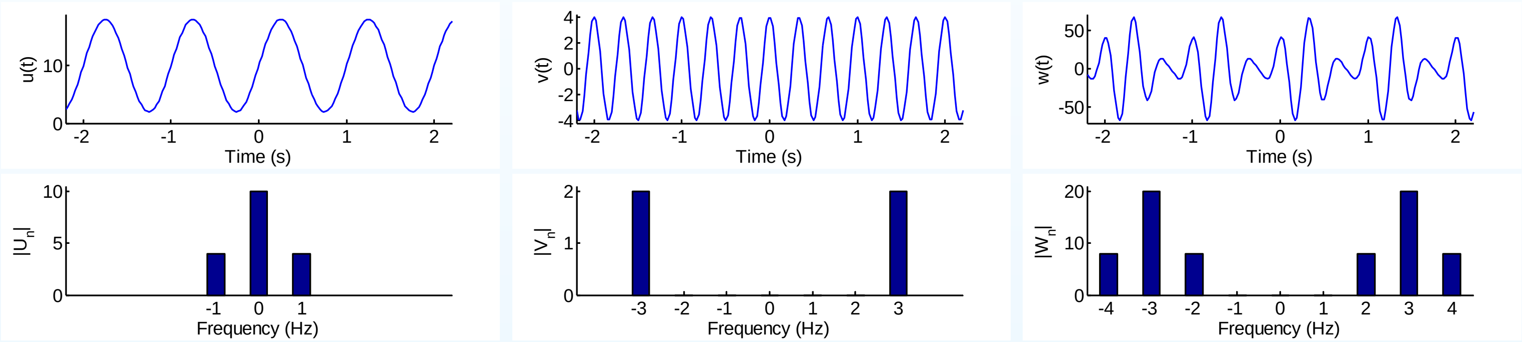

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To convolve U_n and V_n:

Replace each harmonic in V_n by a scaled copy of the entire $\{U_n\}$ and sum the complex-valued coefficients of any overlapping harmonics.

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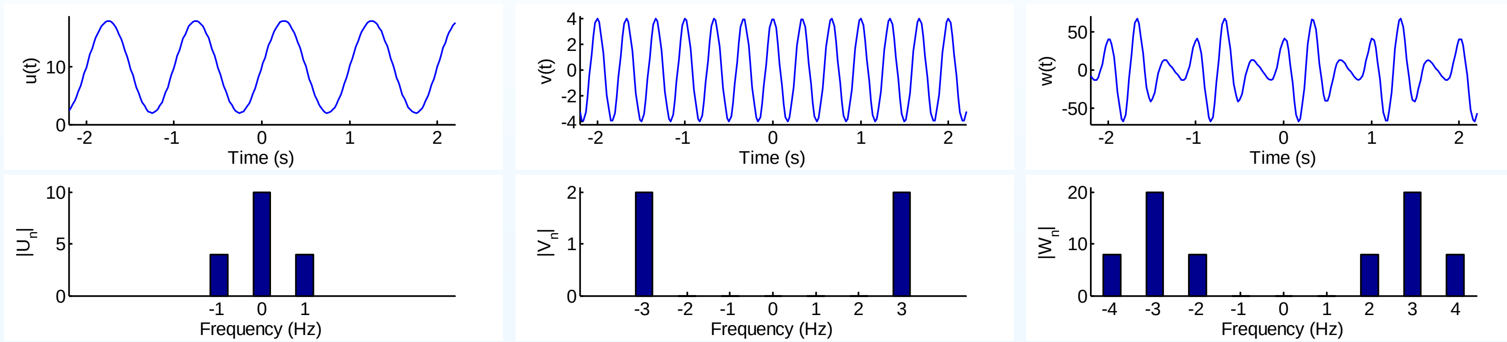

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To convolve U_n and V_n:

Replace each harmonic in V_n by a scaled copy of the entire $\{U_n\}$ (or vice versa) and sum the complex-valued coefficients of any overlapping harmonics.

Convolution and Polynomial Multiplication

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The coefficient of x^r consists of all the coefficient pair from U and V where the subscripts add up to r.

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The coefficient of x^r consists of all the coefficient pair from U and V where the subscripts add up to r. For example, for $r = 3$:

$$W_3 = U_3V_0 + U_2V_1 + U_1V_2$$

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Convolution and Polynomial Multiplication

Two polynomials: $u(x) = U_3x^3 + U_2x^2 + U_1x + U_0$
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Now multiply the two polynomials together:

$$\begin{aligned} w(x) &= u(x)v(x) \\ &= U_3V_2x^5 + (U_3V_1 + U_2V_2)\,x^4 + (U_3V_0 + U_2V_1 + U_1V_2)\,x^3 \\ &\quad + (U_2V_0 + U_1V_1 + U_0V_2)\,x^2 + (U_1V_0 + U_0V_1)\,x + U_0V_0 \end{aligned}$$

The coefficient of x^r consists of all the coefficient pair from U and V where the subscripts add up to r. For example, for $r = 3$:

$$W_3 = U_3V_0 + U_2V_1 + U_1V_2 = \sum_{m=0}^{2} U_{3-m}V_m$$

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$$u(t) = \sum_{n=-\infty}^{\infty} U_n e^{i2\pi nFt} = \sum_{n=-\infty}^{\infty} U_n \left(e^{i2\pi Ft}\right)^n$$

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