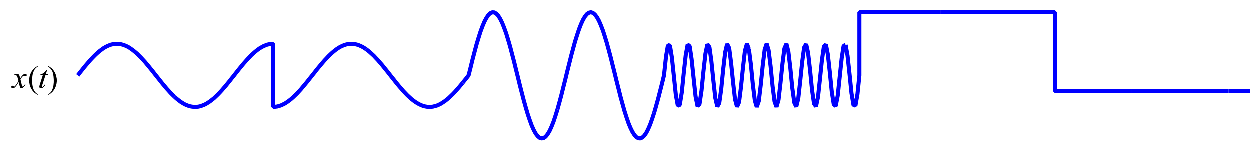
16: Transients (B)

Piecewise steady state inputs

We will consider input signals that are sinusoidal or constant for a particular time interval and then suddenly change in amplitude, phase or frequency.



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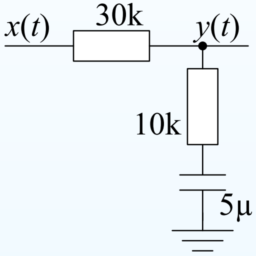
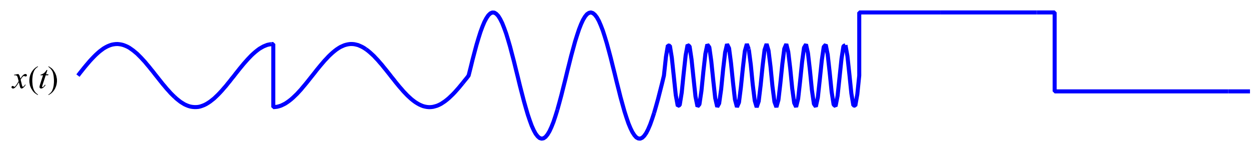


[only one C or L]



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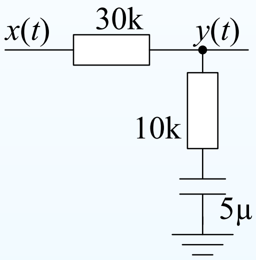

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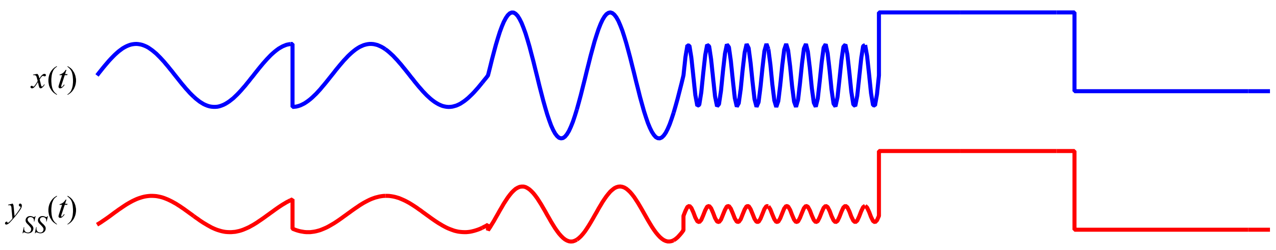

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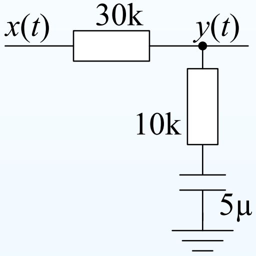

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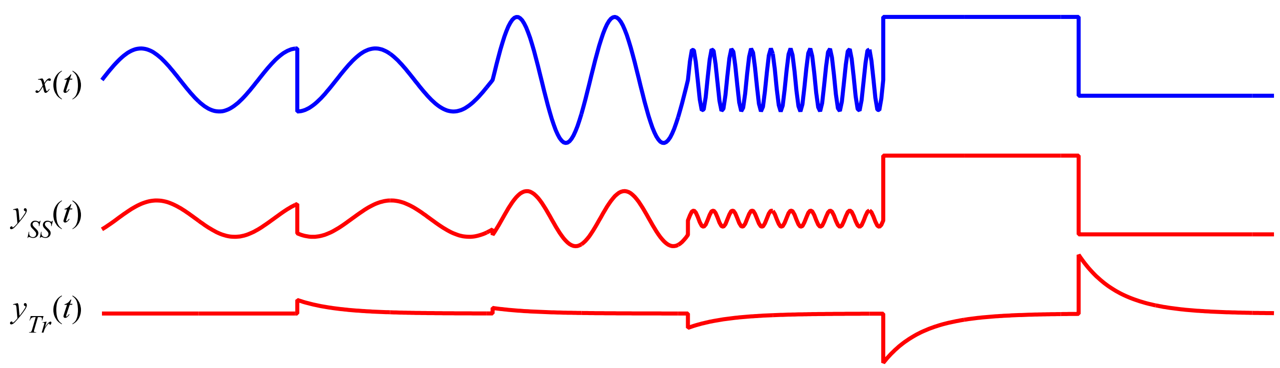

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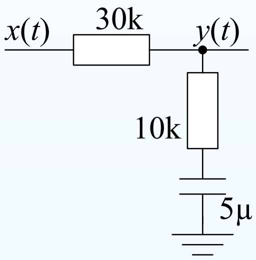

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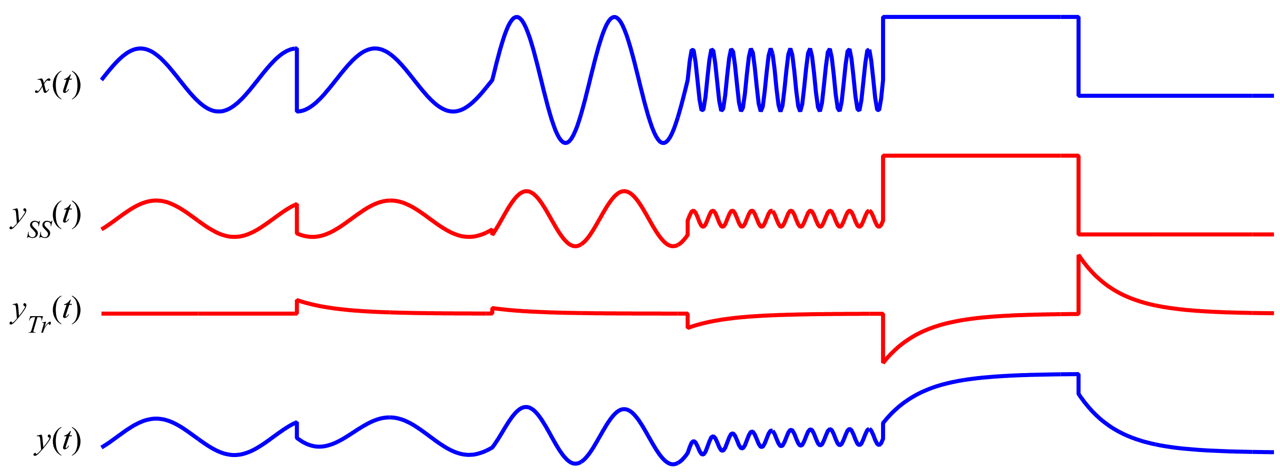
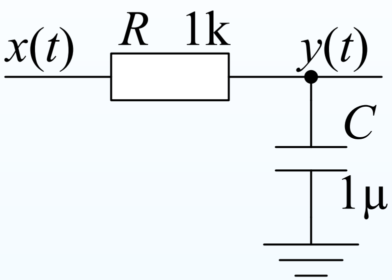


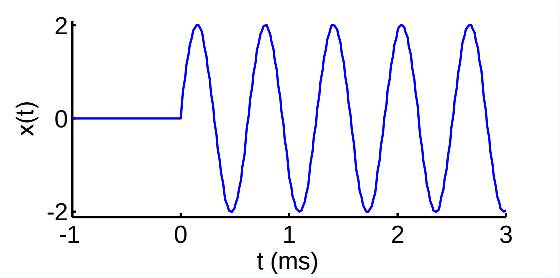
Sinusoidal Input

For $t < 0$: $y(t) = x(t) = 0$

For $t \geq 0$: $x = 2\sin \omega t \Rightarrow X = -2j$

$\tau = RC = 1\,\text{ms}, \omega = 10\,\text{krad/s}$





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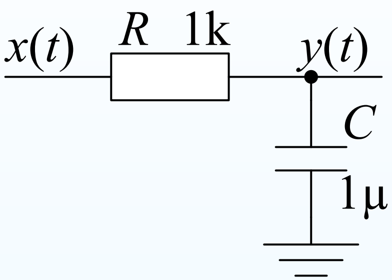

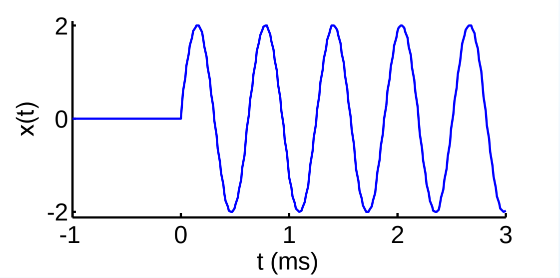

Steady State (for $t \geq 0$)

$$\frac{Y}{X} = \frac{1}{j\omega RC+1} = 0.1\angle -84°$$

Sinusoidal Input

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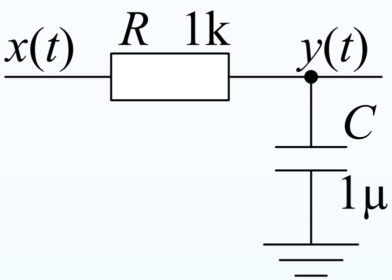

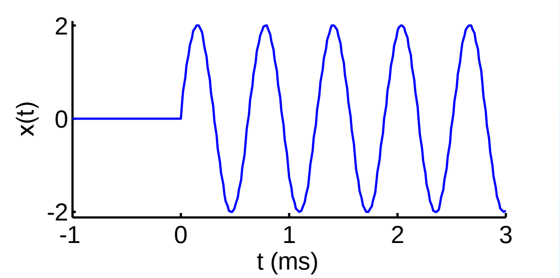

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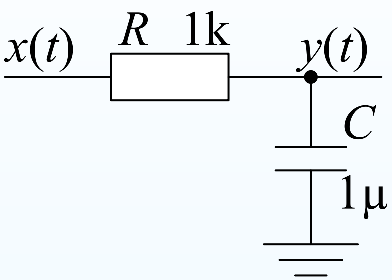

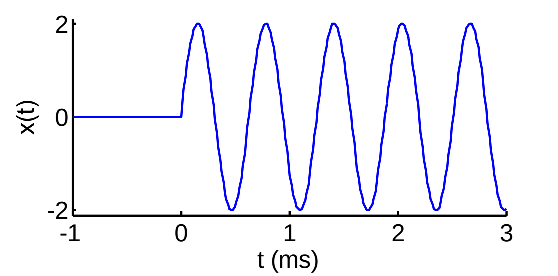

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$y_{SS}(t) = 0.2\cos\left(\omega t - 174°\right)$

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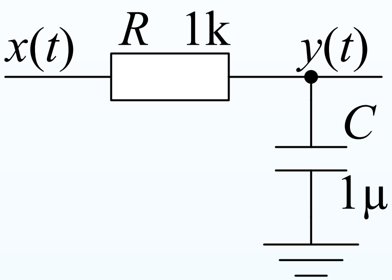

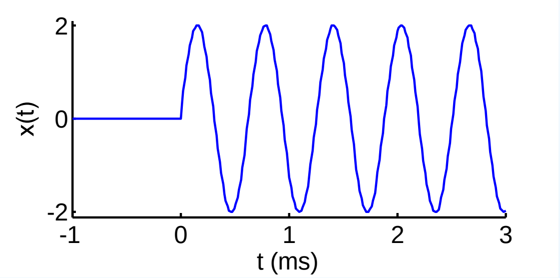

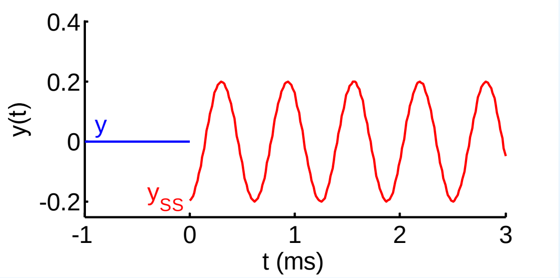

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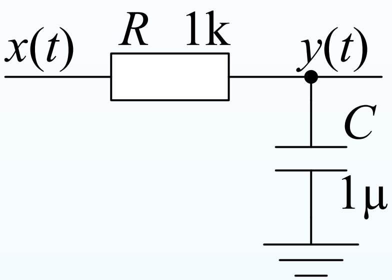

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Steady State + Transient

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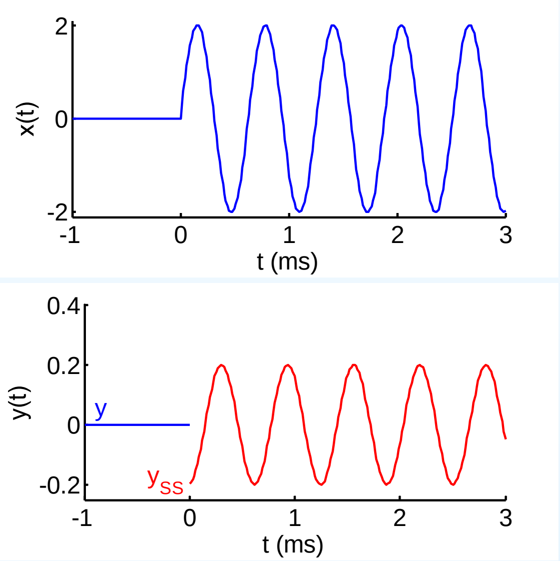

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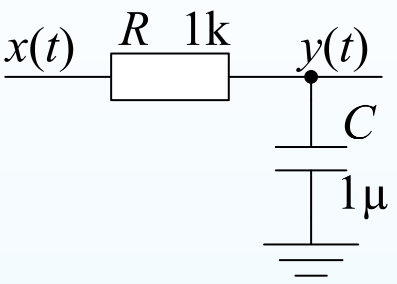

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Steady State + Transient

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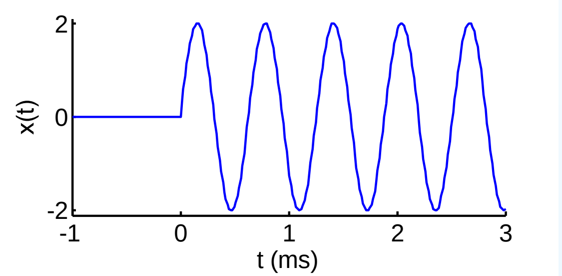
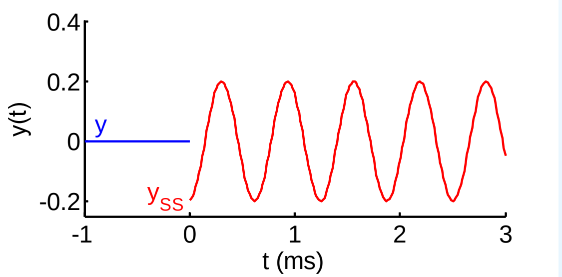
Transient Amplitude

$$y(0+) = 0.2\cos(-174°) + A$$





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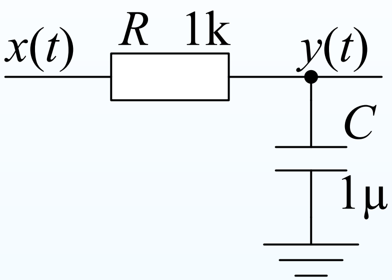

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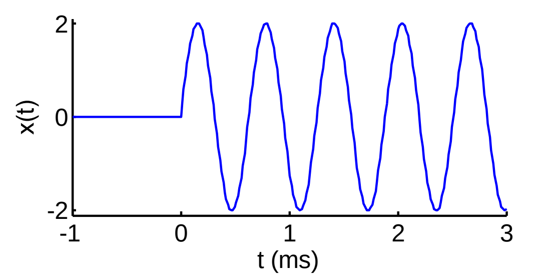

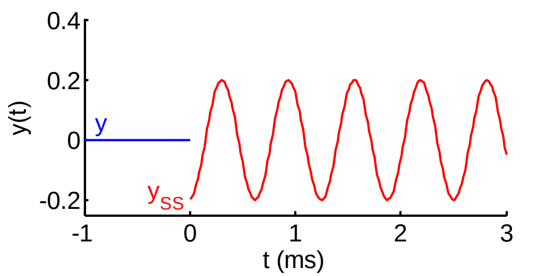

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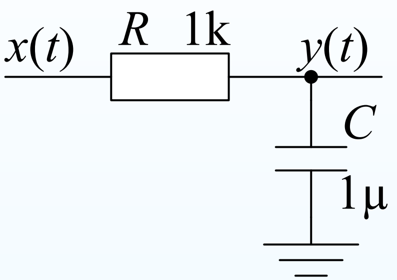

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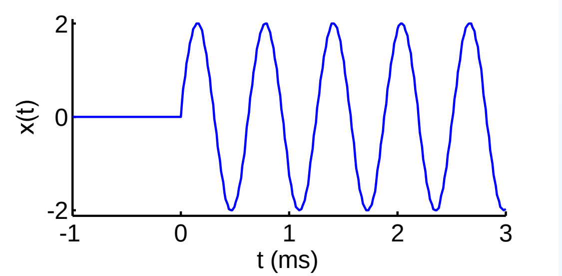

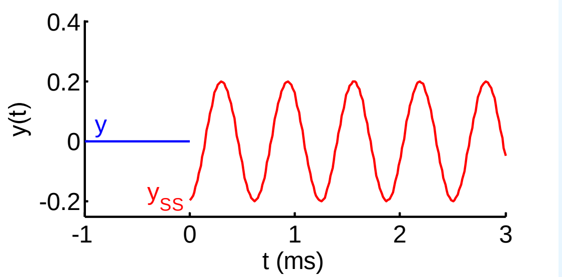

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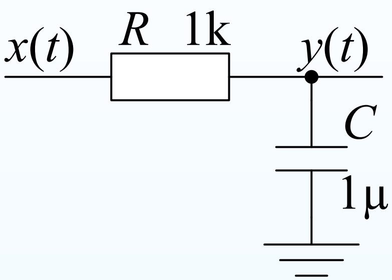

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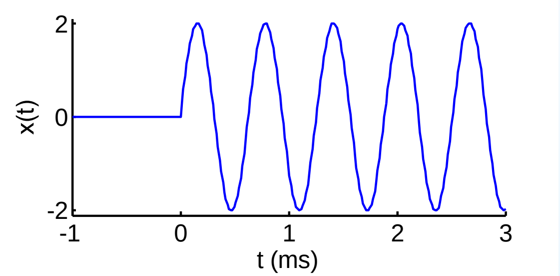

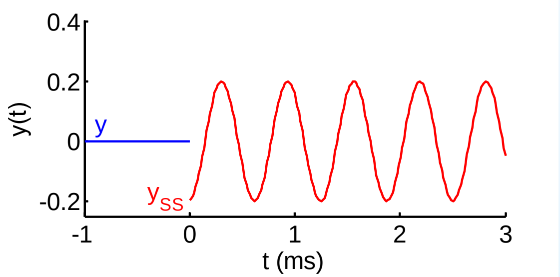

Steady State + Transient

$y(t) = 0.2\cos\left(\omega t - 174°\right) + Ae^{-t/\tau}$

Transient Amplitude

$$y(0+) = 0.2\cos\left(-174°\right) + A$$
$$= -0.198 + A$$

$$y(0+) = y(0-) = 0 \Rightarrow A = 0.198$$

Sinusoidal Input

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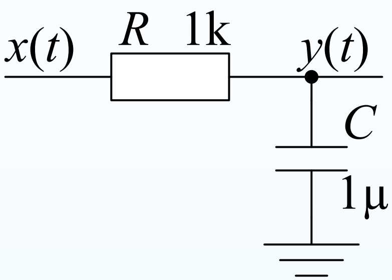

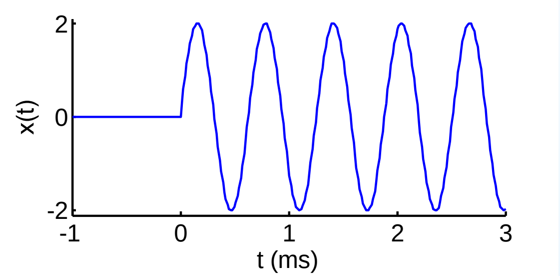

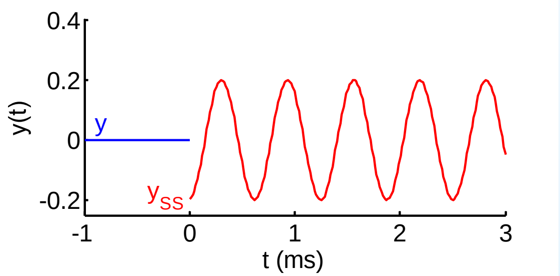

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$$y_{SS}(t) = 0.2\cos(\omega t - 174°)$$

Steady State + Transient

$$y(t) = 0.2\cos(\omega t - 174°) + Ae^{-t/\tau}$$

Transient Amplitude

$$y(0+) = 0.2\cos(-174°) + A$$
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$$y(0+) = y(0-) = 0 \Rightarrow A = 0.198 \Rightarrow y_{Tr}(t) = 0.198e^{-t/\tau}$$

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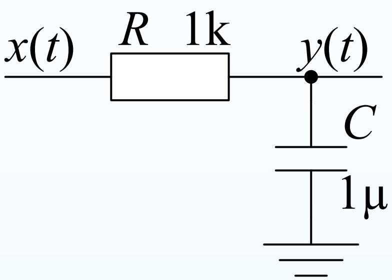

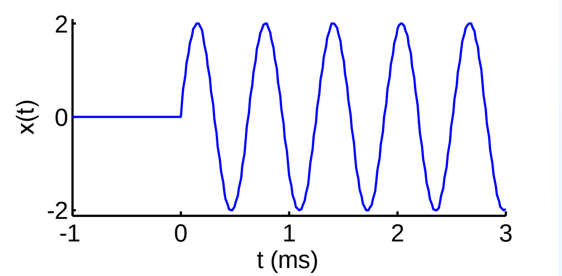

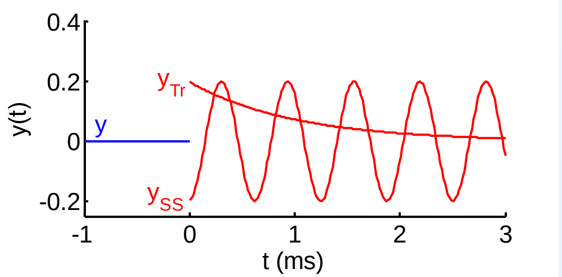

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Steady State + Transient

$$y(t) = 0.2\cos\left(\omega t - 174^\circ\right) + Ae^{-t/\tau}$$

Transient Amplitude

$$y(0+) = 0.2\cos\left(-174^\circ\right) + A$$

$$= -0.198 + A$$

$$y(0+) = y(0-) = 0 \Rightarrow A = 0.198 \Rightarrow y_{Tr}(t) = 0.198e^{-t/\tau}$$

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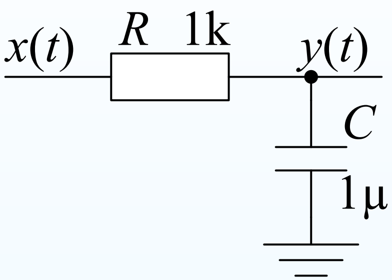

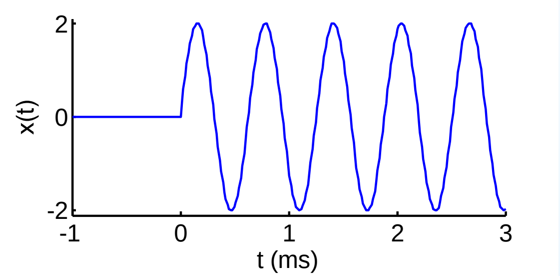

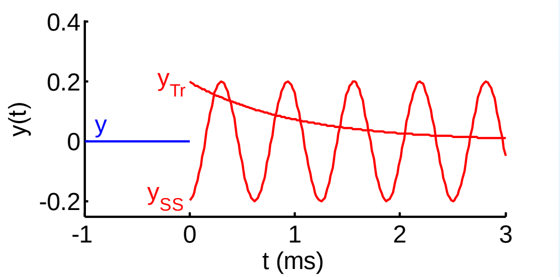

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Steady State + Transient

$$y(t) = 0.2\cos\left(\omega t - 174^\circ\right) + Ae^{-t/\tau}$$

Transient Amplitude

$$y(0+) = 0.2\cos\left(-174^\circ\right) + A$$

$$= -0.198 + A$$

$$y(0+) = y(0-) = 0 \Rightarrow A = 0.198 \Rightarrow y_{Tr}(t) = 0.198e^{-t/\tau}$$

Complete Expression for $y(t)$

$$y(t) = 0.2\cos\left(\omega t - 174^\circ\right) + 0.198e^{-t/\tau}$$

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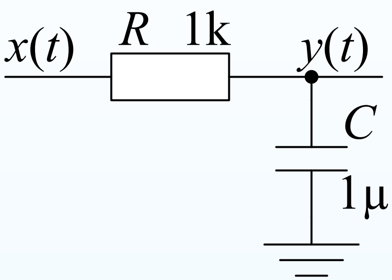
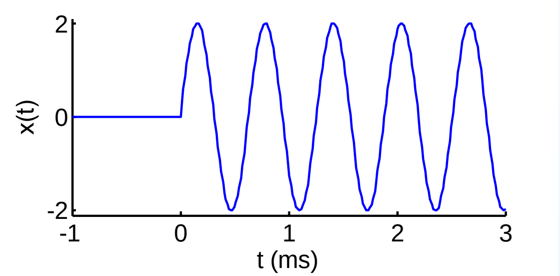
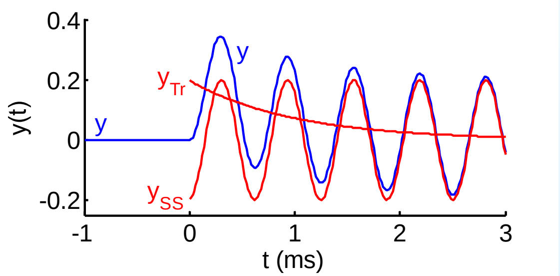
$\tau = RC = 1\,\text{ms}, \omega = 10\,\text{krad/s}$







Steady State (for $t \geq 0$)

$$\frac{Y}{X} = \frac{1}{j\omega RC+1} = 0.1\angle -84°$$

$$Y = X \times \frac{Y}{X} = -2j \times 0.1\angle -84°$$

$$y_{SS}(t) = 0.2\cos(\omega t - 174°)$$

Steady State + Transient

$$y(t) = 0.2\cos(\omega t - 174°) + Ae^{-t/\tau}$$

Transient Amplitude

$$y(0+) = 0.2\cos(-174°) + A$$
$$= -0.198 + A$$

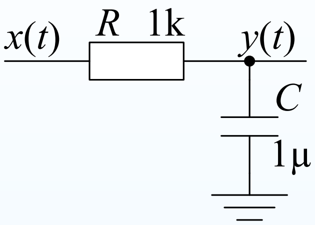
$$y(0+) = y(0-) = 0 \Rightarrow A = 0.198 \Rightarrow y_{Tr}(t) = 0.198e^{-t/\tau}$$

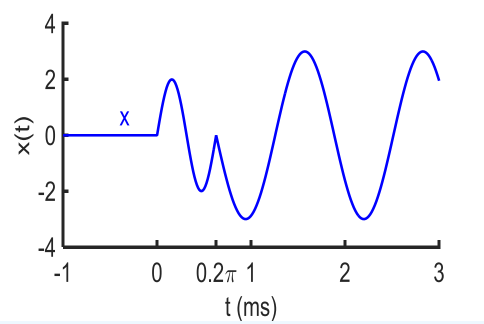
Complete Expression for $y(t)$

$$y(t) = 0.2\cos(\omega t - 174°) + 0.198e^{-t/\tau}$$

Multiple Discontinuities

For $0 \le t < 0.2\pi$ ms: $X = -2j, \omega_1 = 10\,\text{k}, \tau = 1\,\text{ms}$
prev page $\Rightarrow y(t) = 0.2\cos(\omega t - 174°) + 0.198e^{-t/\tau}$





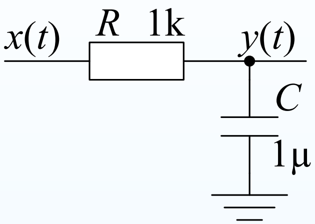
Multiple Discontinuities

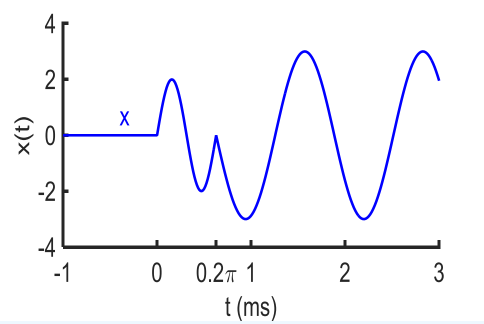
For $0 \le t < 0.2\pi$ ms: $X = -2j, \omega_1 = 10\,\text{k}, \tau = 1\,\text{ms}$
prev page $\Rightarrow y(t) = 0.2\cos(\omega t - 174°) + 0.198e^{-t/\tau}$

Steady State (for $t \ge 0.0002\pi = 0.63\,\text{ms}$)

$X = -3j, \omega_2 = 5\,\text{k}$

$\frac{Y}{X} = \frac{1}{j\omega_2 RC+1} = 0.2\angle - 79°$





Multiple Discontinuities

For $0 \le t < 0.2\pi$ ms: $X = -2j, \omega_1 = 10\,\text{k}, \tau = 1\,\text{ms}$
prev page $\Rightarrow y(t) = 0.2\cos(\omega t - 174°) + 0.198e^{-t/\tau}$

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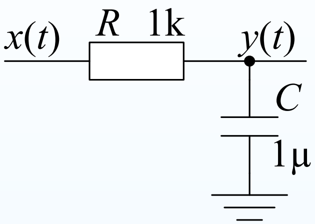
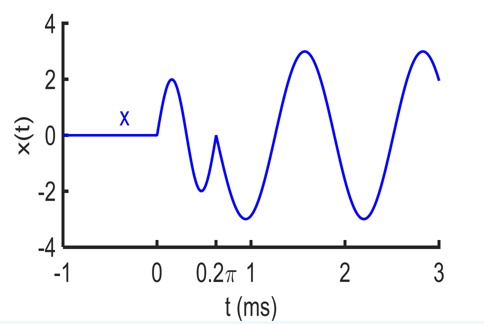
$X = -3j, \omega_2 = 5\,\text{k}$

$\frac{Y}{X} = \frac{1}{j\omega_2 RC+1} = 0.2\angle -79°$

$Y = X \times \frac{Y}{X} = -3j \times 0.2\angle -79°$





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For $0 \le t < 0.2\pi$ ms: $X = -2j, \omega_1 = 10\,\text{k}, \tau = 1\,\text{ms}$
prev page $\Rightarrow y(t) = 0.2\cos(\omega t - 174°) + 0.198e^{-t/\tau}$

Steady State (for $t \ge 0.0002\pi = 0.63\,\text{ms}$)

$X = -3j, \omega_2 = 5\,\text{k}$
$\frac{Y}{X} = \frac{1}{j\omega_2 RC+1} = 0.2\angle -79°$
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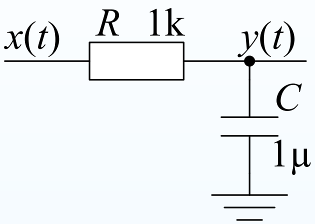
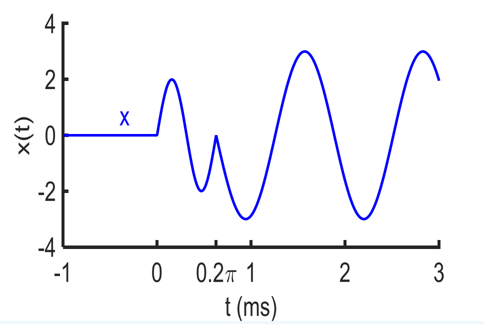
$y_{SS}(t) = 0.59\cos(\omega_2 t - 169°)$





Multiple Discontinuities

For $0 \le t < 0.2\pi$ ms: $X = -2j, \omega_1 = 10\,\text{k}, \tau = 1\,\text{ms}$

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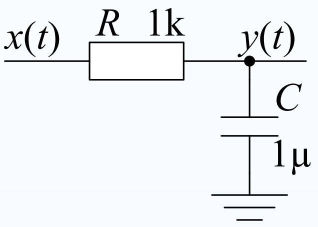

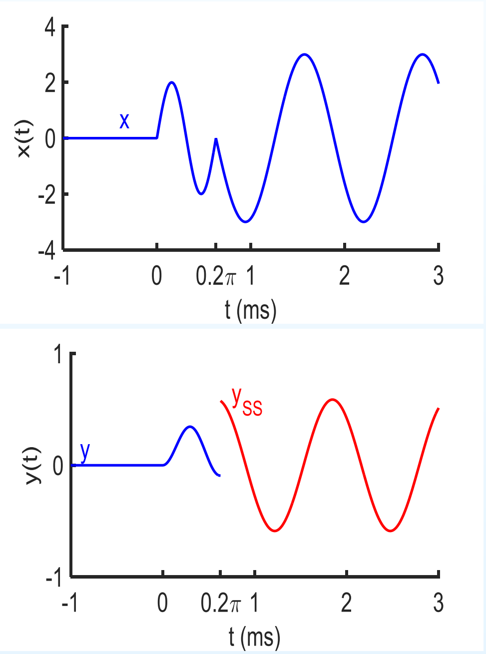

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For $0 \le t < 0.2\pi$ ms: $X = -2j, \omega_1 = 10\,\text{k}, \tau = 1\,\text{ms}$

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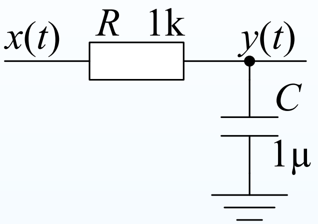

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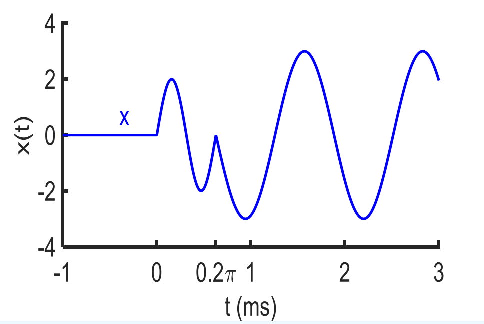
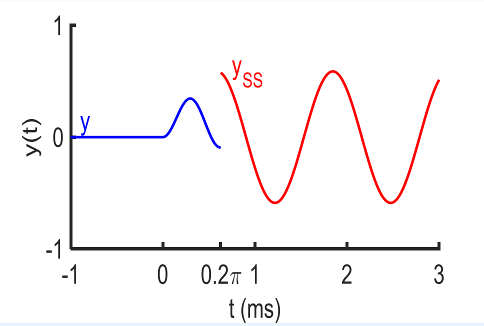
$y_{SS}(t) = 0.59\cos(\omega_2 t - 169°)$



Steady State + Transient (for $t \ge 0.63\,\text{ms}$)

$y = 0.59\cos(\omega_2 t - 169°) + Be^{-(t-0.00063)/\tau}$



Multiple Discontinuities

For $0 \le t < 0.2\pi$ ms: $X = -2j, \omega_1 = 10\,\text{k}, \tau = 1\,\text{ms}$
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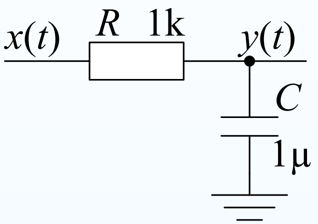

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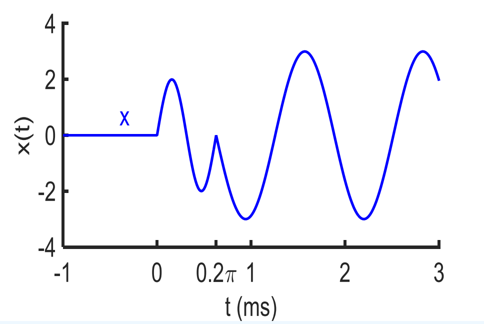

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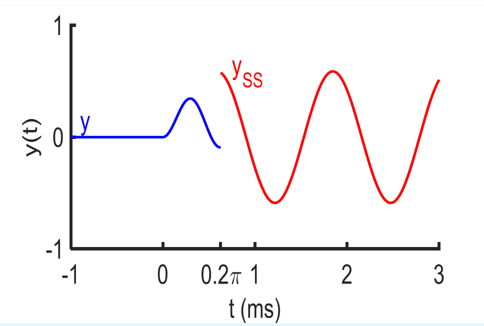


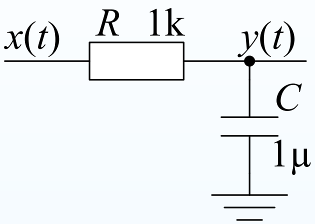
Transient Amplitude (at $t = 0.63\,\text{ms}$)

$y(0.00063+) = 0.59\cos(0.00063\omega_2 - 169°) + B$

Multiple Discontinuities

For $0 \le t < 0.2\pi$ ms: $X = -2j, \omega_1 = 10\,\text{k}, \tau = 1\,\text{ms}$

prev page $\Rightarrow y(t) = 0.2\cos(\omega t - 174°) + 0.198e^{-t/\tau}$



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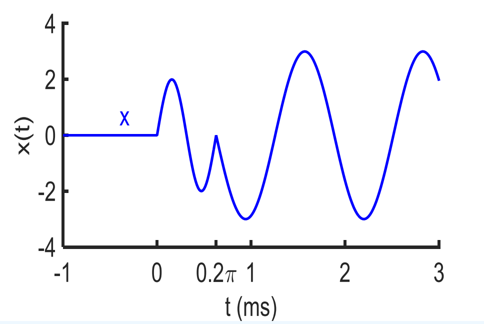
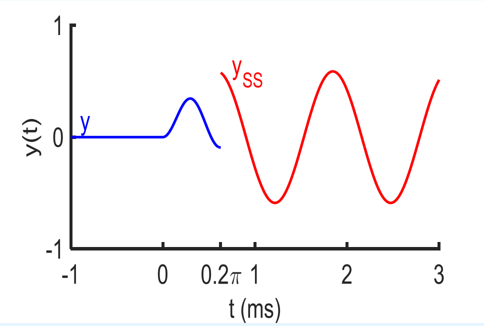
$y_{SS}(t) = 0.59\cos(\omega_2 t - 169°)$



Steady State + Transient (for $t \ge 0.63\,\text{ms}$)

$y = 0.59\cos(\omega_2 t - 169°) + Be^{-(t-0.00063)/\tau}$



Transient Amplitude (at $t = 0.63\,\text{ms}$)

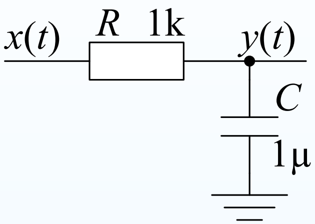
$y(0.00063+) = 0.59\cos(0.00063\omega_2 - 169°) + B$

$= 0.577 + B$

Multiple Discontinuities

For $0 \leq t < 0.2\pi$ ms: $X = -2j, \omega_1 = 10\,\text{k}, \tau = 1\,\text{ms}$

prev page $\Rightarrow y(t) = 0.2\cos(\omega t - 174°) + 0.198e^{-t/\tau}$



Steady State (for $t \geq 0.0002\pi = 0.63\,\text{ms}$)

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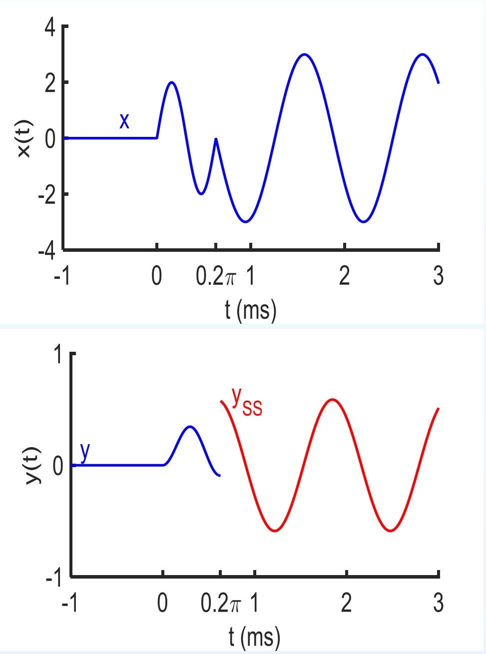


Steady State + Transient (for $t \geq 0.63\,\text{ms}$)

$$y = 0.59\cos(\omega_2 t - 169°) + Be^{-(t-0.00063)/\tau}$$

Transient Amplitude (at $t = 0.63\,\text{ms}$)

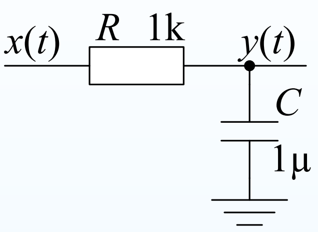
$$y(0.00063+) = 0.59\cos(0.00063\omega_2 - 169°) + B$$
$$= 0.577 + B$$

$$y(0.00063-) = 0.2\cos(0.00063\omega_1 - 174°) + 0.198e^{-0.00063/\tau} = -0.092$$

Multiple Discontinuities

For $0 \le t < 0.2\pi$ ms: $X = -2j,\ \omega_1 = 10\,\text{k},\ \tau = 1\,\text{ms}$

prev page $\Rightarrow y(t) = 0.2\cos(\omega t - 174°) + 0.198e^{-t/\tau}$



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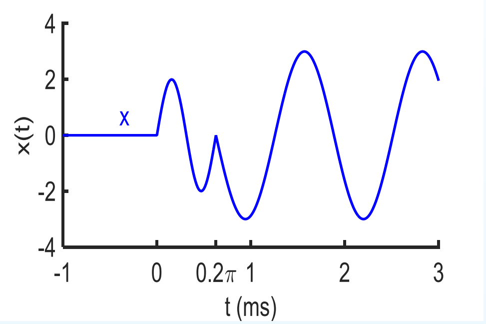
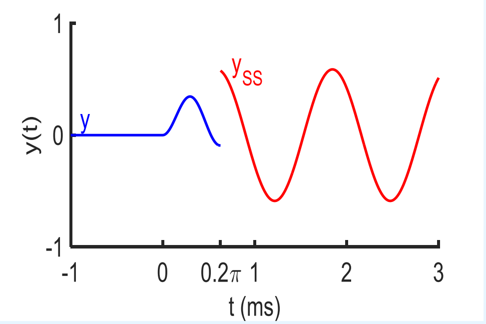


Steady State + Transient (for $t \ge 0.63\,\text{ms}$)

$$y = 0.59\cos(\omega_2 t - 169°) + Be^{-(t-0.00063)/\tau}$$



Transient Amplitude (at $t = 0.63\,\text{ms}$)

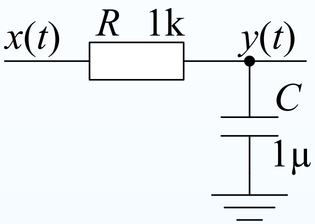
$$y(0.00063+) = 0.59\cos(0.00063\omega_2 - 169°) + B$$
$$= 0.577 + B$$

$$y(0.00063-) = 0.2\cos(0.00063\omega_1 - 174°) + 0.198e^{-0.00063/\tau} = -0.092$$
$$\Rightarrow 0.577 + B = -0.092 \Rightarrow B = -0.67$$

Multiple Discontinuities

For $0 \le t < 0.2\pi$ ms: $X = -2j, \omega_1 = 10\,\text{k}, \tau = 1\,\text{ms}$

prev page $\Rightarrow y(t) = 0.2\cos(\omega t - 174°) + 0.198e^{-t/\tau}$



Steady State (for $t \ge 0.0002\pi = 0.63\,\text{ms}$)

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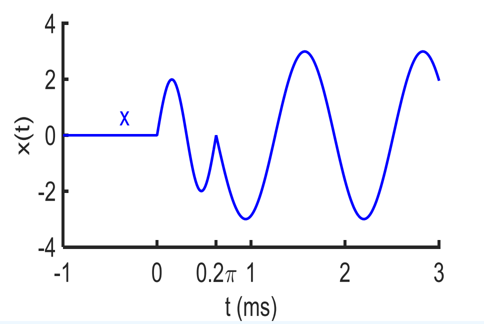
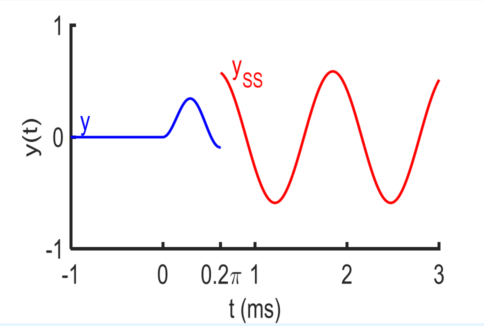


Steady State + Transient (for $t \ge 0.63\,\text{ms}$)

$y = 0.59\cos(\omega_2 t - 169°) + Be^{-(t-0.00063)/\tau}$



Transient Amplitude (at $t = 0.63\,\text{ms}$)

$y(0.00063+) = 0.59\cos(0.00063\omega_2 - 169°) + B$

$= 0.577 + B$

$y(0.00063-) = 0.2\cos(0.00063\omega_1 - 174°) + 0.198e^{-0.00063/\tau} = -0.092$

$\Rightarrow 0.577 + B = -0.092 \Rightarrow B = -0.67 \Rightarrow y_{Tr} = -0.67e^{-(t-0.00063)/\tau}$

Multiple Discontinuities

For $0 \le t < 0.2\pi$ ms: $X = -2j, \omega_1 = 10\,\text{k}, \tau = 1\,\text{ms}$

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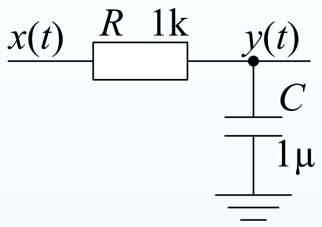

Steady State (for $t \ge 0.0002\pi = 0.63\,\text{ms}$)

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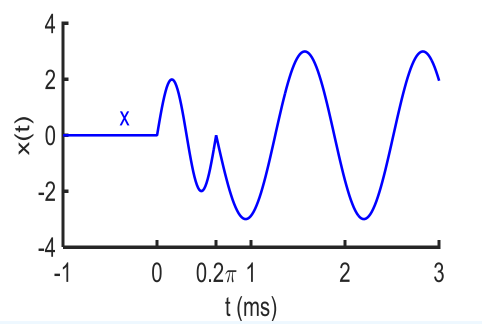
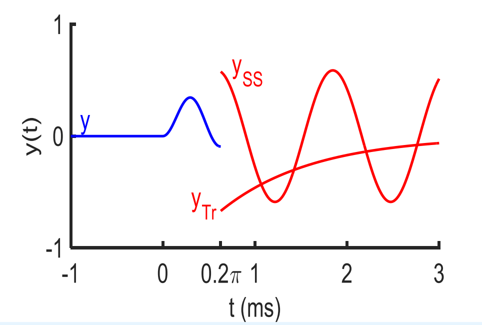


Steady State + Transient (for $t \ge 0.63\,\text{ms}$)

$$y = 0.59\cos(\omega_2 t - 169°) + Be^{-(t-0.00063)/\tau}$$



Transient Amplitude (at $t = 0.63\,\text{ms}$)

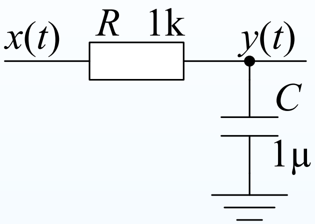
$$y(0.00063+) = 0.59\cos(0.00063\omega_2 - 169°) + B$$
$$= 0.577 + B$$

$$y(0.00063-) = 0.2\cos(0.00063\omega_1 - 174°) + 0.198e^{-0.00063/\tau} = -0.092$$
$$\Rightarrow 0.577 + B = -0.092 \Rightarrow B = -0.67 \Rightarrow y_{Tr} = -0.67e^{-(t-0.00063)/\tau}$$

Multiple Discontinuities

For $0 \le t < 0.2\pi$ ms: $X = -2j, \omega_1 = 10\,\text{k}, \tau = 1\,\text{ms}$

prev page $\Rightarrow y(t) = 0.2\cos(\omega t - 174°) + 0.198e^{-t/\tau}$



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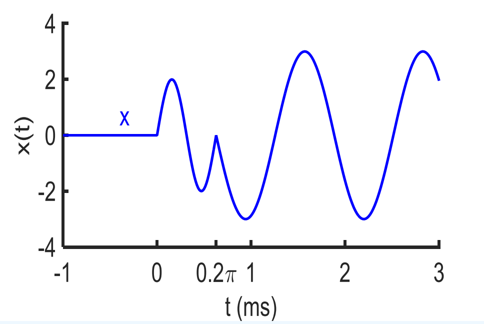
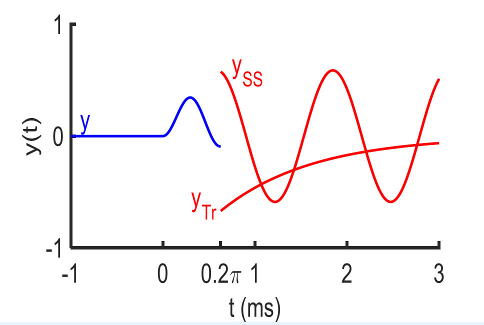


Steady State + Transient (for $t \ge 0.63\,\text{ms}$)

$$y = 0.59\cos(\omega_2 t - 169°) + Be^{-(t-0.00063)/\tau}$$

Transient Amplitude (at $t = 0.63\,\text{ms}$)

$$y(0.00063+) = 0.59\cos(0.00063\omega_2 - 169°) + B$$
$$= 0.577 + B$$



$$y(0.00063-) = 0.2\cos(0.00063\omega_1 - 174°) + 0.198e^{-0.00063/\tau} = -0.092$$
$$\Rightarrow 0.577 + B = -0.092 \Rightarrow B = -0.67 \Rightarrow y_{Tr} = -0.67e^{-(t-0.00063)/\tau}$$

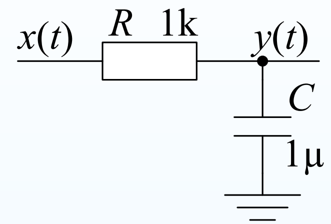
Complete Expression for $y(t)$ (for $t \ge 0.63\,\text{ms}$)

$$y(t) = 0.59\cos(\omega_2 t - 169°) - 0.67e^{-(t-0.00063)/\tau}$$

Multiple Discontinuities

For $0 \le t < 0.2\pi$ ms: $X = -2j, \omega_1 = 10\,\text{k}, \tau = 1\,\text{ms}$

prev page $\Rightarrow y(t) = 0.2\cos(\omega t - 174°) + 0.198e^{-t/\tau}$



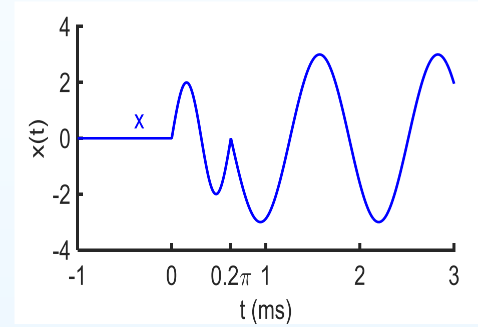
Steady State (for $t \ge 0.0002\pi = 0.63\,\text{ms}$)

$$X = -3j, \omega_2 = 5\,\text{k}$$

$$\frac{Y}{X} = \frac{1}{j\omega_2 RC + 1} = 0.2\angle -79°$$

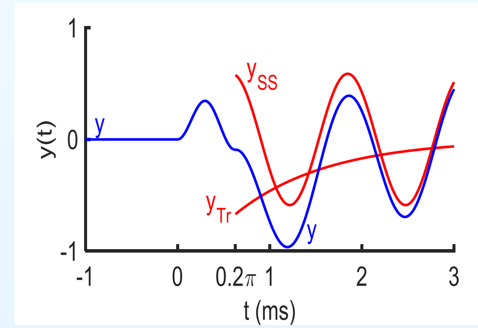
$$Y = X \times \frac{Y}{X} = -3j \times 0.2\angle -79°$$

$$y_{SS}(t) = 0.59\cos(\omega_2 t - 169°)$$



Steady State + Transient (for $t \ge 0.63\,\text{ms}$)

$$y = 0.59\cos(\omega_2 t - 169°) + Be^{-(t-0.00063)/\tau}$$



Transient Amplitude (at $t = 0.63\,\text{ms}$)

$$y(0.00063+) = 0.59\cos(0.00063\omega_2 - 169°) + B$$
$$= 0.577 + B$$

$$y(0.00063-) = 0.2\cos(0.00063\omega_1 - 174°) + 0.198e^{-0.00063/\tau} = -0.092$$

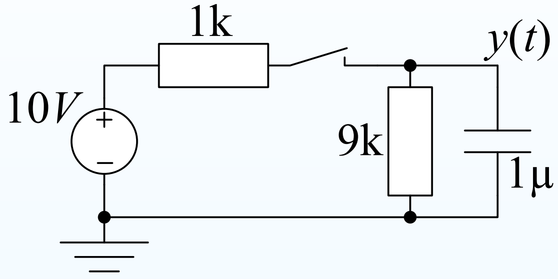
$$\Rightarrow 0.577 + B = -0.092 \Rightarrow B = -0.67 \Rightarrow y_{Tr} = -0.67e^{-(t-0.00063)/\tau}$$

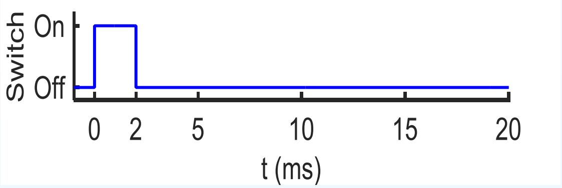
Complete Expression for $y(t)$ (for $t \ge 0.63\,\text{ms}$)

$$y(t) = 0.59\cos(\omega_2 t - 169°) - 0.67e^{-(t-0.00063)/\tau}$$

Switched Circuit

Operating the switch changes τ:

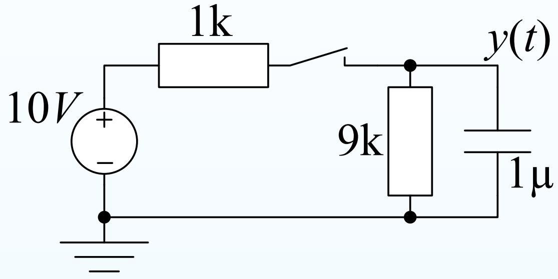


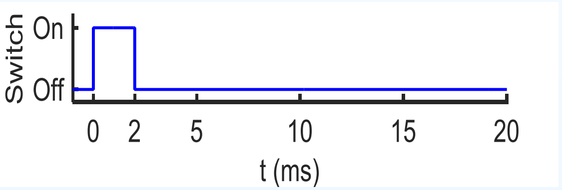


Switched Circuit

Operating the switch changes τ:

Closed: $\tau_C = (1\,\text{k}\,||\,9\,\text{k}) \times C = 0.9\,\text{ms}$



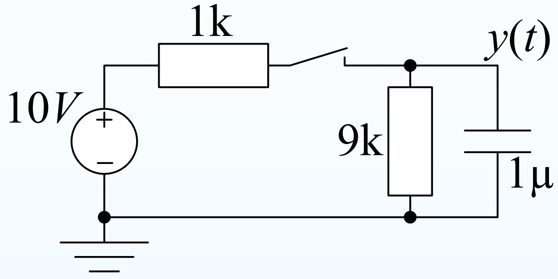


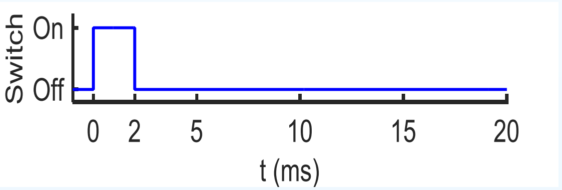
Switched Circuit

Operating the switch changes τ:

Closed: $\tau_C = (1\,\text{k}\,||\,9\,\text{k}) \times C = 0.9\,\text{ms}$

Open: $\tau_O = 9\,\text{k} \times C = 9\,\text{ms}$





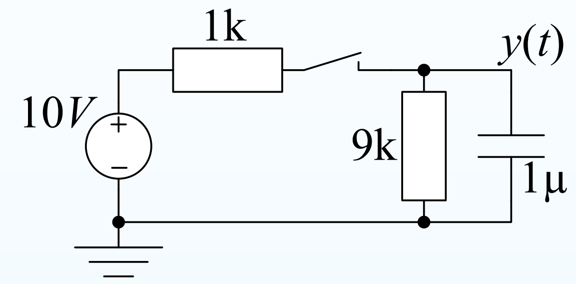
Switched Circuit

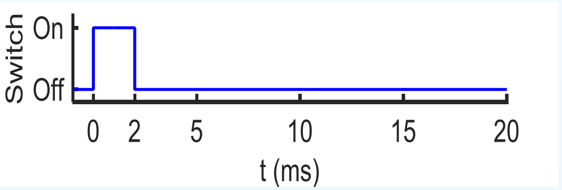
Operating the switch changes τ:

Closed: $\tau_C = (1\,\mathrm{k}\,||\,9\,\mathrm{k}) \times C = 0.9\,\mathrm{ms}$

Open: $\tau_O = 9\,\mathrm{k} \times C = 9\,\mathrm{ms}$

Switch closed at $t = 0$.





Switched Circuit

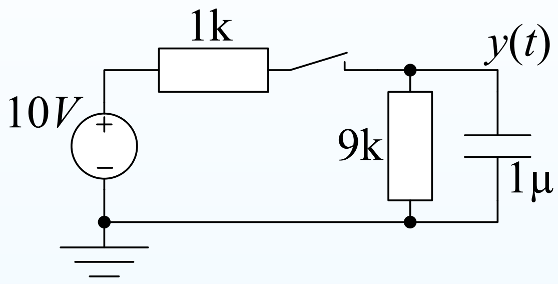
Operating the switch changes τ:

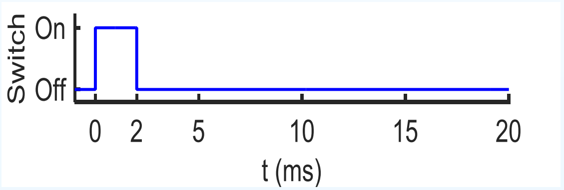
Closed: $\tau_C = (1\,\text{k}\,||\,9\,\text{k}) \times C = 0.9\,\text{ms}$

Open: $\tau_O = 9\,\text{k} \times C = 9\,\text{ms}$

Switch closed at $t = 0$.

$$y_{SS} = 10 \times \frac{9}{10} = 9\,\text{V}$$



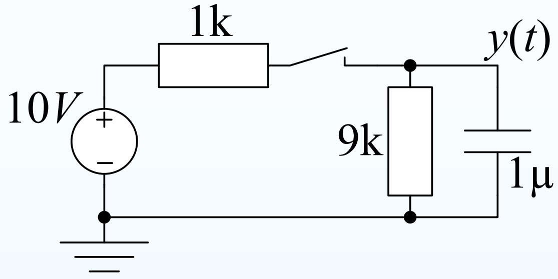


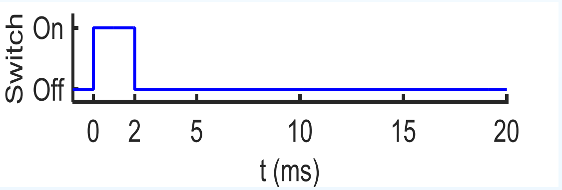
Switched Circuit

Operating the switch changes τ:

Closed: $\tau_C = (1\,\mathrm{k}\,||\,9\,\mathrm{k}) \times C = 0.9\,\mathrm{ms}$

Open: $\tau_O = 9\,\mathrm{k} \times C = 9\,\mathrm{ms}$





Switch closed at $t = 0$.

$$y_{SS} = 10 \times \frac{9}{10} = 9\,\mathrm{V}$$

$$y(t) = 9 - 9e^{-t/\tau_C}$$

Switched Circuit

Operating the switch changes τ:

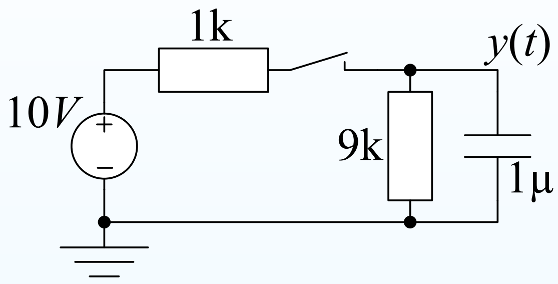
Closed: $\tau_C = (1\,\text{k} \,||\, 9\,\text{k}) \times C = 0.9\,\text{ms}$

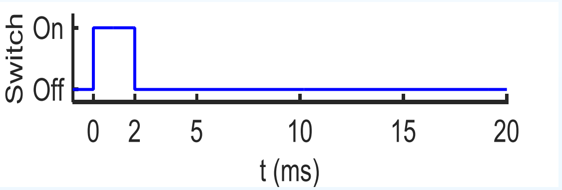
Open: $\tau_O = 9\,\text{k} \times C = 9\,\text{ms}$

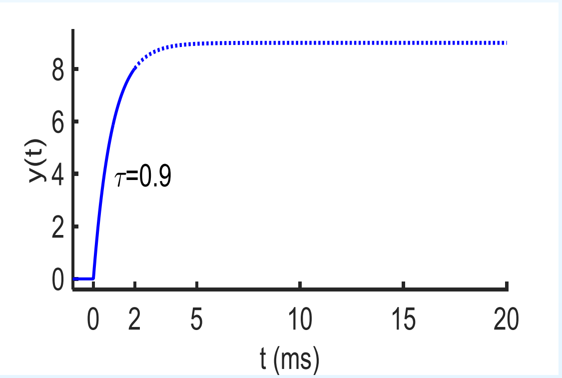
Switch closed at $t = 0$.

$$y_{SS} = 10 \times \frac{9}{10} = 9\,\text{V}$$

$$y(t) = 9 - 9e^{-t/\tau_C}$$







Switched Circuit

Operating the switch changes τ:

Closed: $\tau_C = (1\,\text{k}\,||\,9\,\text{k}) \times C = 0.9\,\text{ms}$

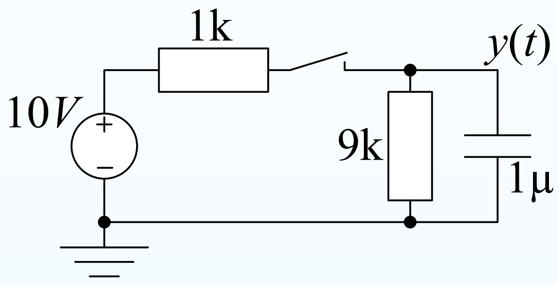
Open: $\tau_O = 9\,\text{k} \times C = 9\,\text{ms}$

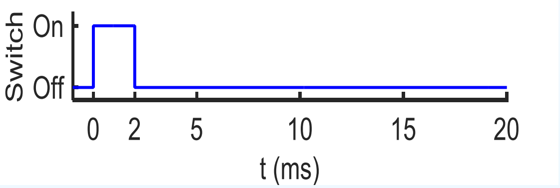
Switch closed at $t = 0$.

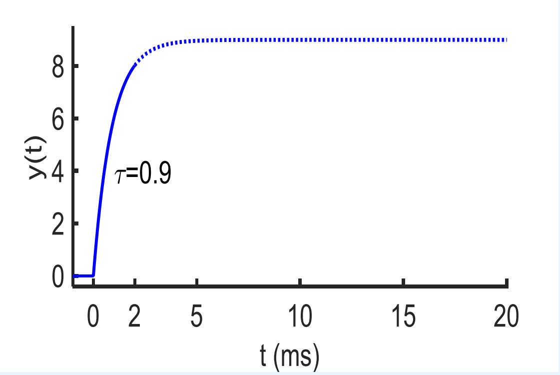
$y_{SS} = 10 \times \frac{9}{10} = 9\,\text{V}$

$y(t) = 9 - 9e^{-t/\tau_C}$

$y(2-) = 9 - 9e^{-2/0.9} = 8.02$







Switched Circuit

Operating the switch changes τ:

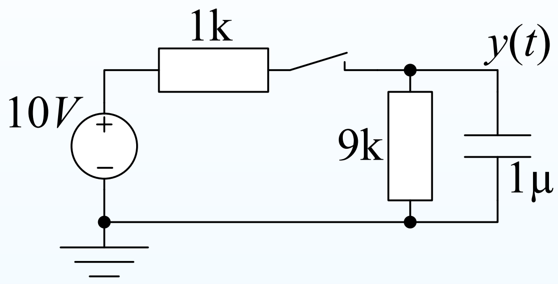
Closed: $\tau_C = (1\,\text{k}\,||\,9\,\text{k}) \times C = 0.9\,\text{ms}$

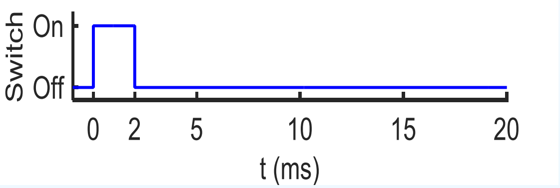
Open: $\tau_O = 9\,\text{k} \times C = 9\,\text{ms}$

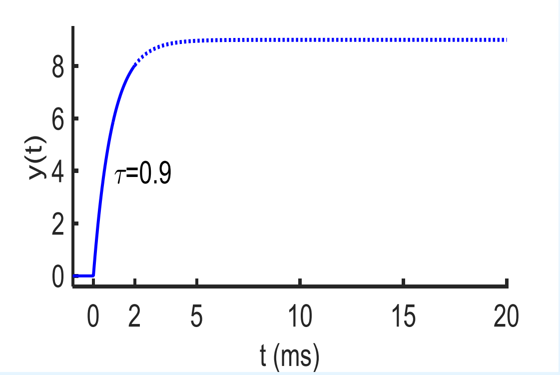
Switch closed at $t = 0$.

$$y_{SS} = 10 \times \frac{9}{10} = 9\,\text{V}$$
$$y(t) = 9 - 9e^{-t/\tau_C}$$
$$y(2-) = 9 - 9e^{-2/0.9} = 8.02$$

Switch opened at $t = 2$.







Switched Circuit

Operating the switch changes τ:

Closed: $\tau_C = (1\,\text{k} \,||\, 9\,\text{k}) \times C = 0.9\,\text{ms}$

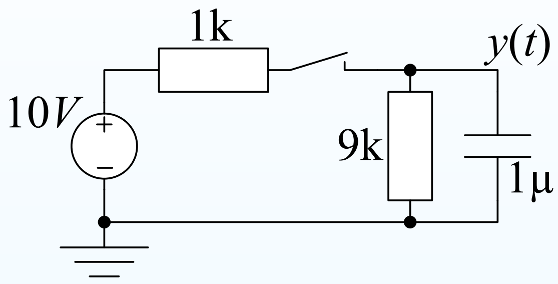
Open: $\tau_O = 9\,\text{k} \times C = 9\,\text{ms}$

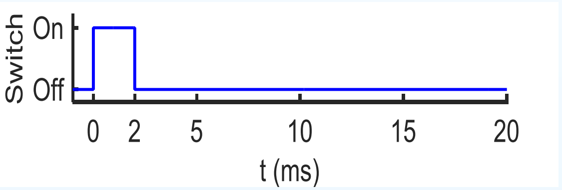
Switch closed at $t = 0$.

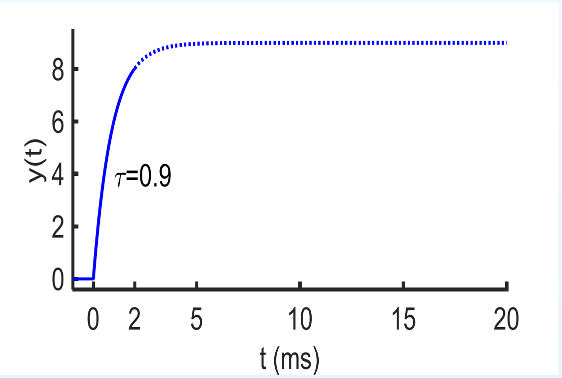
$$y_{SS} = 10 \times \frac{9}{10} = 9\,\text{V}$$
$$y(t) = 9 - 9e^{-t/\tau_C}$$
$$y(2-) = 9 - 9e^{-2/0.9} = 8.02$$

Switch opened at $t = 2$.

$$y_{SS} = 0\,\text{V}$$
$$y(t) = 0 + Ae^{-(t-2)/\tau_O}$$







Switched Circuit

Operating the switch changes τ:

Closed: $\tau_C = (1\,\text{k}\,||\,9\,\text{k}) \times C = 0.9\,\text{ms}$

Open: $\tau_O = 9\,\text{k} \times C = 9\,\text{ms}$

Switch closed at $t = 0$.

$$y_{SS} = 10 \times \frac{9}{10} = 9\,\text{V}$$
$$y(t) = 9 - 9e^{-t/\tau_C}$$
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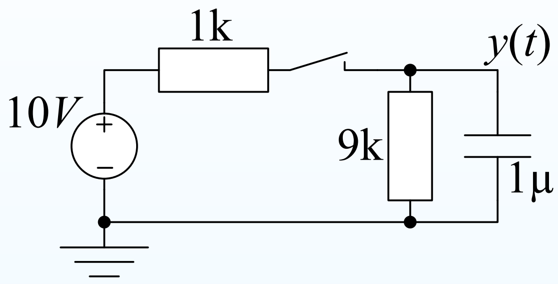
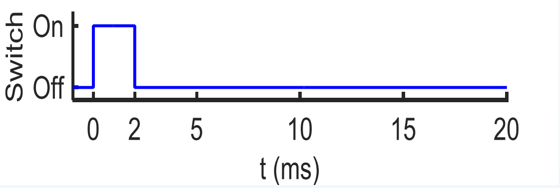
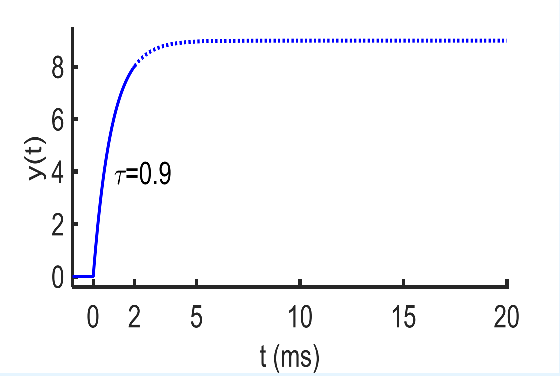
Switch opened at $t = 2$.

$$y_{SS} = 0\,\text{V}$$
$$y(t) = 0 + Ae^{-(t-2)/\tau_O}$$
$$y(2+) = A = y(2-) = 8.02$$







Switched Circuit

Operating the switch changes τ:

Closed: $\tau_C = (1\,\text{k}\,||\,9\,\text{k}) \times C = 0.9\,\text{ms}$

Open: $\tau_O = 9\,\text{k} \times C = 9\,\text{ms}$

Switch closed at $t = 0$.

$$y_{SS} = 10 \times \frac{9}{10} = 9\,\text{V}$$
$$y(t) = 9 - 9e^{-t/\tau_C}$$
$$y(2-) = 9 - 9e^{-2/0.9} = 8.02$$

Switch opened at $t = 2$.

$$y_{SS} = 0\,\text{V}$$
$$y(t) = 0 + Ae^{-(t-2)/\tau_O}$$
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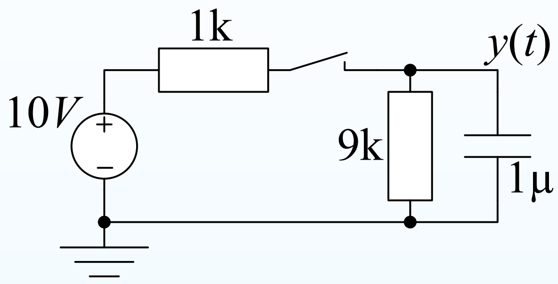
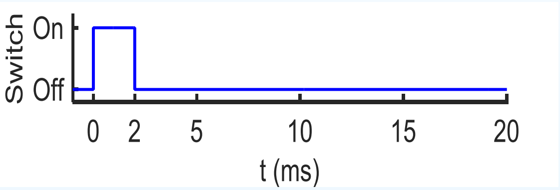
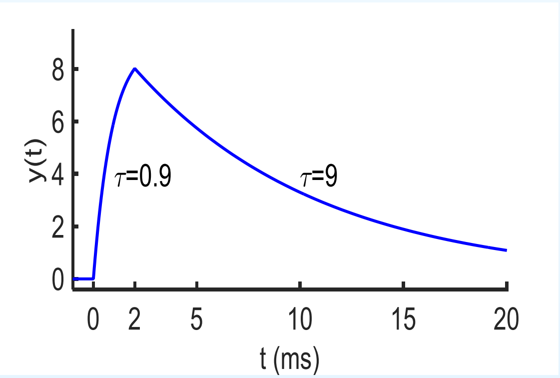




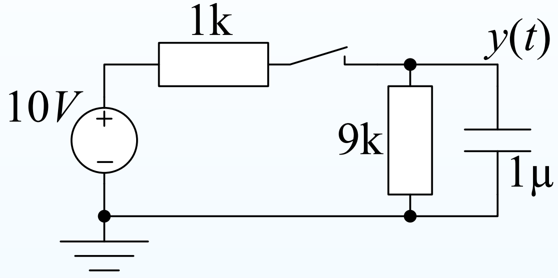


Switched Circuit

Operating the switch changes τ:

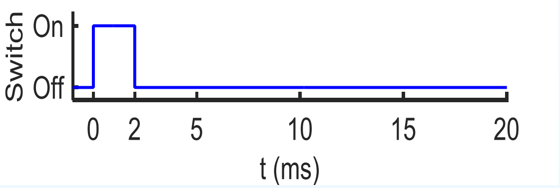
Closed: $\tau_C = (1\,\text{k}\,||\,9\,\text{k}) \times C = 0.9\,\text{ms}$

Open: $\tau_O = 9\,\text{k} \times C = 9\,\text{ms}$



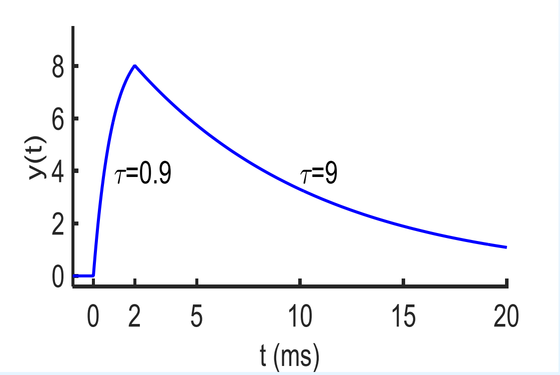
Switch closed at $t = 0$.

$$y_{SS} = 10 \times \frac{9}{10} = 9\,\text{V}$$
$$y(t) = 9 - 9e^{-t/\tau_C}$$
$$y(2-) = 9 - 9e^{-2/0.9} = 8.02$$



Switch opened at $t = 2$.

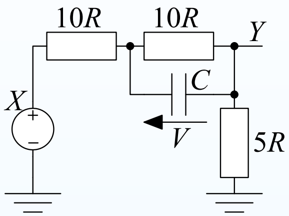
$$y_{SS} = 0\,\text{V}$$
$$y(t) = 0 + Ae^{-(t-2)/\tau_O}$$
$$y(2+) = A = y(2-) = 8.02$$
$$y(20) = 8.02e^{-(20-2)/9} = 1.09$$



Transfer Function

Phasor nodal analysis:

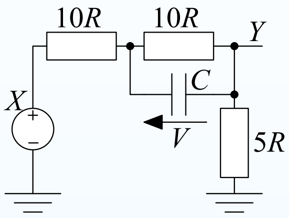
$$\frac{Y}{X} = \frac{5R}{15R+\frac{10R}{1+10j\omega RC}} = \frac{10j\omega RC+1}{30j\omega RC+5}$$



Transfer Function

Phasor nodal analysis:

$$\frac{Y}{X} = \frac{5R}{15R+\frac{10R}{1+10j\omega RC}} = \frac{10j\omega RC+1}{30j\omega RC+5} = 0.2\frac{\frac{j\omega}{p}+1}{\frac{j\omega}{q}+1}$$

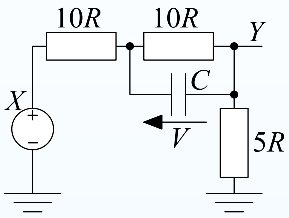


Transfer Function

Phasor nodal analysis:

$$\frac{Y}{X} = \frac{5R}{15R+\frac{10R}{1+10j\omega RC}} = \frac{10j\omega RC+1}{30j\omega RC+5} = 0.2\frac{\frac{j\omega}{p}+1}{\frac{j\omega}{q}+1}$$

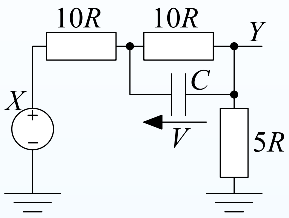
Corner freqencies: $p = \frac{1}{10RC}, q = \frac{1}{6RC}$



Transfer Function

Phasor nodal analysis:

$$\frac{Y}{X} = \frac{5R}{15R + \frac{10R}{1+10j\omega RC}} = \frac{10j\omega RC+1}{30j\omega RC+5} = 0.2\frac{\frac{j\omega}{p}+1}{\frac{j\omega}{q}+1}$$



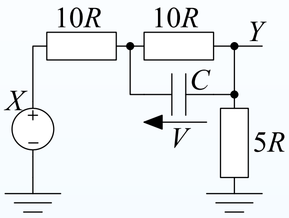
Corner freqencies: $p = \frac{1}{10RC}$, $q = \frac{1}{6RC}$, HF gain = $\frac{1}{3}$

Transfer Function

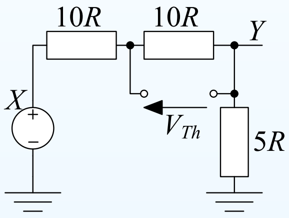
Phasor nodal analysis:

$$\frac{Y}{X} = \frac{5R}{15R+\frac{10R}{1+10j\omega RC}} = \frac{10j\omega RC+1}{30j\omega RC+5} = 0.2\frac{\frac{j\omega}{p}+1}{\frac{j\omega}{q}+1}$$

Corner freqencies: $p = \frac{1}{10RC}$, $q = \frac{1}{6RC}$, HF gain = $\frac{1}{3}$



Thévenin Equivalent driving C:

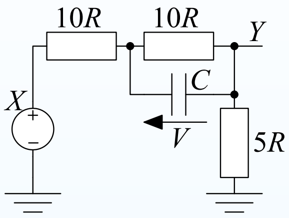


Transfer Function

Phasor nodal analysis:

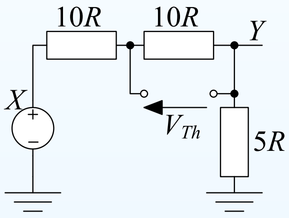
$$\frac{Y}{X} = \frac{5R}{15R+\frac{10R}{1+10j\omega RC}} = \frac{10j\omega RC+1}{30j\omega RC+5} = 0.2\frac{\frac{j\omega}{p}+1}{\frac{j\omega}{q}+1}$$

Corner freqencies: $p = \frac{1}{10RC}$, $q = \frac{1}{6RC}$, HF gain = $\frac{1}{3}$



Thévenin Equivalent driving C:

$$V_{Th} = \frac{2}{5}X,\ R_{Th} = 10R||15R = 6R$$

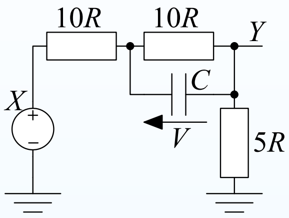


Transfer Function

Phasor nodal analysis:

$$\frac{Y}{X} = \frac{5R}{15R + \frac{10R}{1+10j\omega RC}} = \frac{10j\omega RC+1}{30j\omega RC+5} = 0.2\frac{\frac{j\omega}{p}+1}{\frac{j\omega}{q}+1}$$

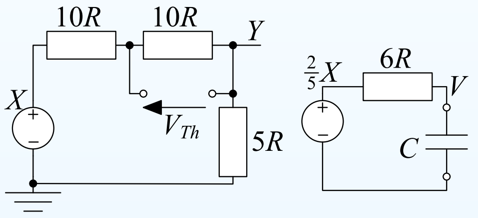
Corner freqencies: $p = \frac{1}{10RC}$, $q = \frac{1}{6RC}$, HF gain = $\frac{1}{3}$



Thévenin Equivalent driving C:

$$V_{Th} = \frac{2}{5}X,\ R_{Th} = 10R||15R = 6R$$

$$V = \frac{2}{5}X \times \frac{1}{6j\omega RC+1}$$

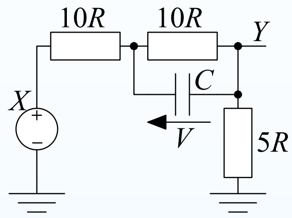


Transfer Function

Phasor nodal analysis:

$$\frac{Y}{X} = \frac{5R}{15R+\frac{10R}{1+10j\omega RC}} = \frac{10j\omega RC+1}{30j\omega RC+5} = 0.2\frac{\frac{j\omega}{p}+1}{\frac{j\omega}{q}+1}$$

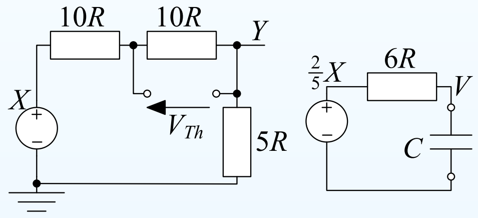
Corner freqencies: $p = \frac{1}{10RC}$, $q = \frac{1}{6RC}$, HF gain = $\frac{1}{3}$



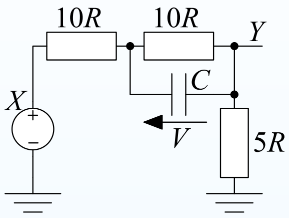
Thévenin Equivalent driving C:

$$V_{Th} = \frac{2}{5}X,\ R_{Th} = 10R||15R = 6R,\ \tau = 6RC$$

$$V = \frac{2}{5}X \times \frac{1}{6j\omega RC+1}$$



Transfer Function



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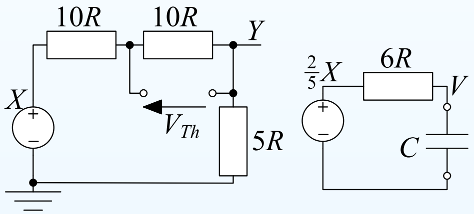

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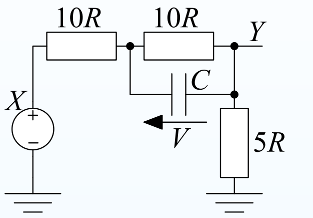

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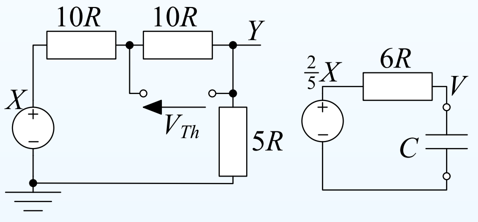

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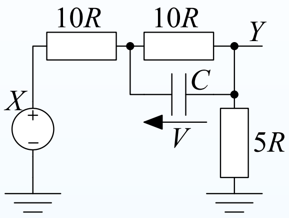

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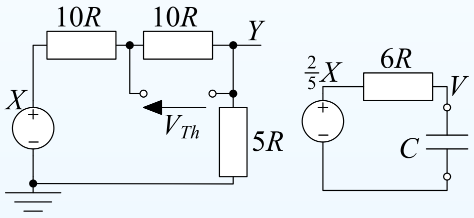

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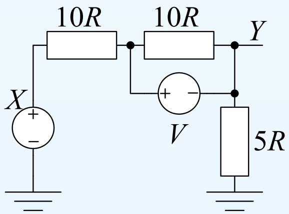

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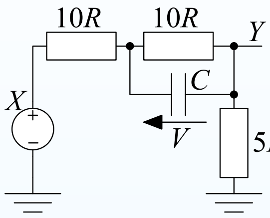

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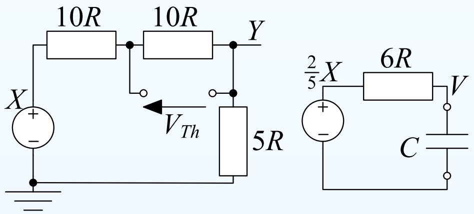

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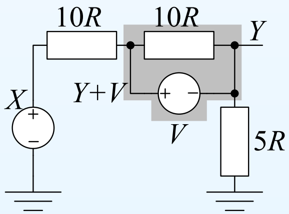

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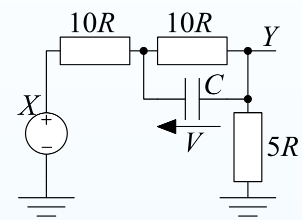

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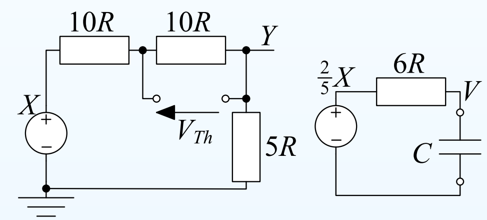

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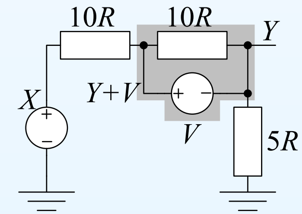

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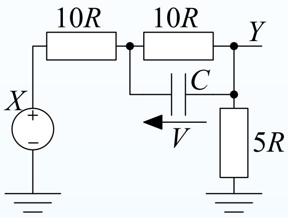

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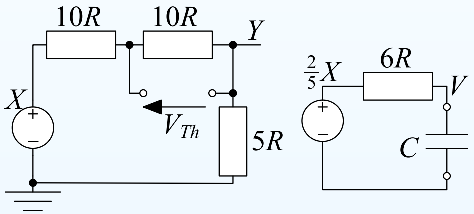

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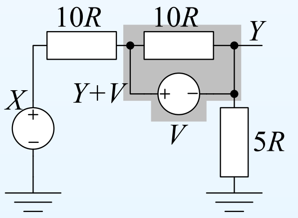

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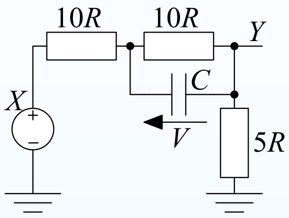

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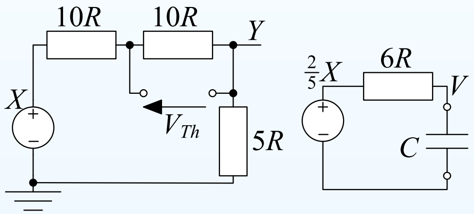

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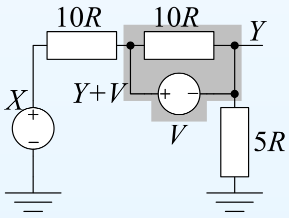

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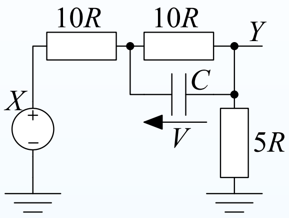

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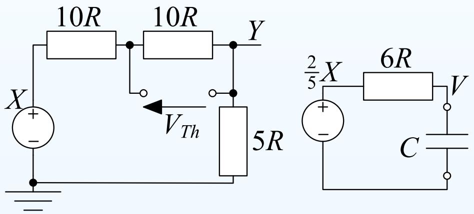

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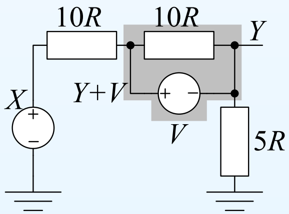

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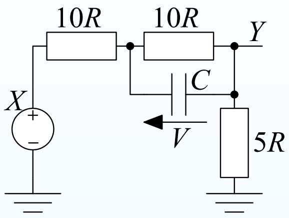

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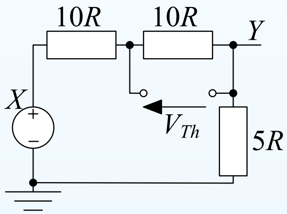

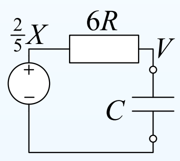

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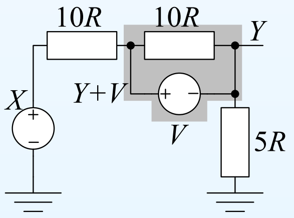

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(1) Denominator corner frequency is always $\frac{1}{\tau}$ for any transfer function in the circuit.

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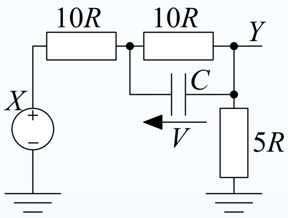

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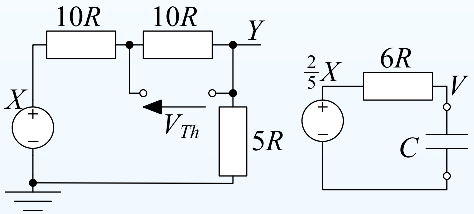

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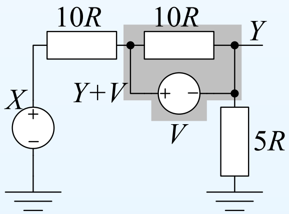

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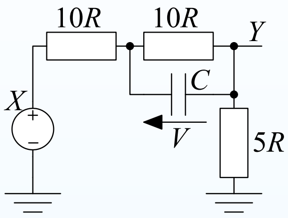

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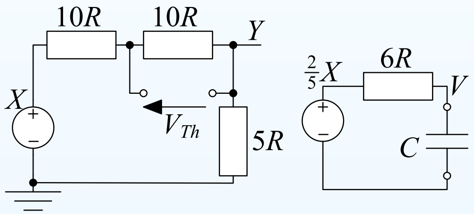

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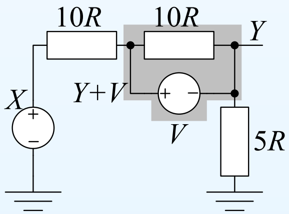

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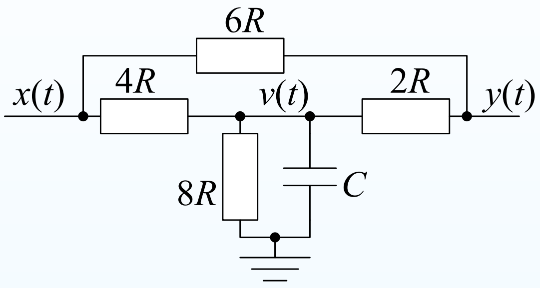
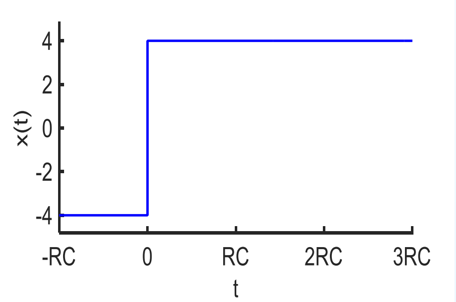
V is never discontinuous so ΔY discontinuity = HF-gain$\times\Delta X$ discontinuity

Transient from Transfer Function

Calculate Transfer Function

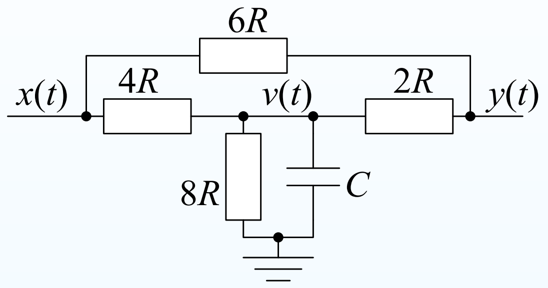


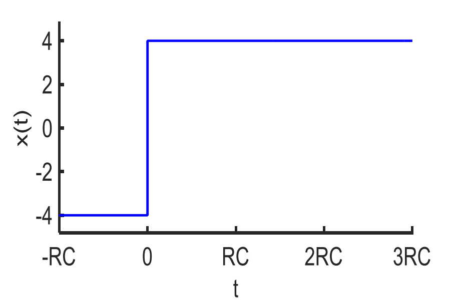


Transient from Transfer Function

Calculate Transfer Function

KCL @ V: $\frac{V-X}{4R}+\frac{V}{8R}+j\omega CV+\frac{V-Y}{2R}=0$





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KCL @ V: $\frac{V-X}{4R} + \frac{V}{8R} + j\omega CV + \frac{V-Y}{2R} = 0$

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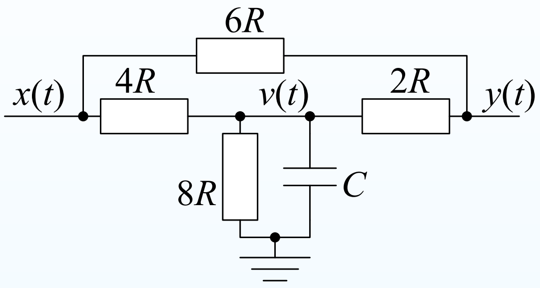
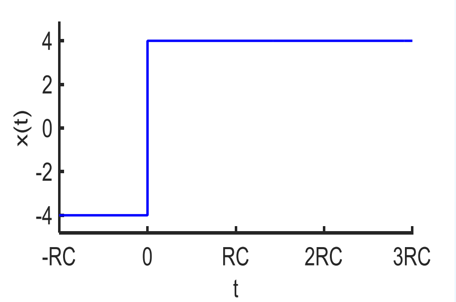




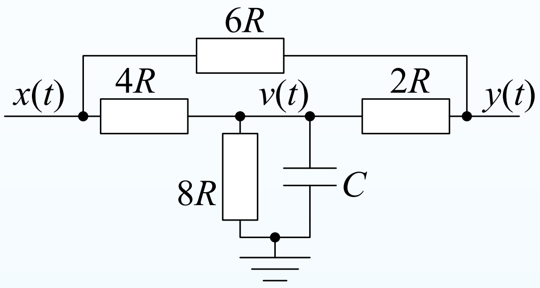
Transient from Transfer Function

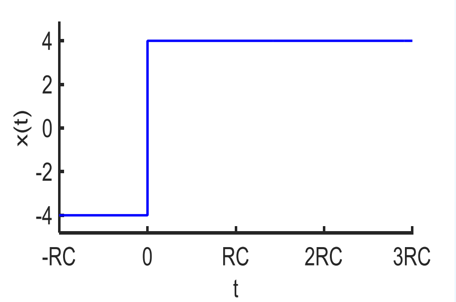
Calculate Transfer Function

KCL @ V: $\frac{V-X}{4R} + \frac{V}{8R} + j\omega CV + \frac{V-Y}{2R} = 0$

KCL @ Y: $\frac{Y-V}{2R} + \frac{Y-X}{6R} = 0$

$\rightarrow$ Transfer Function: $\frac{Y}{X} = \frac{8j\omega RC+13}{32j\omega RC+16}$





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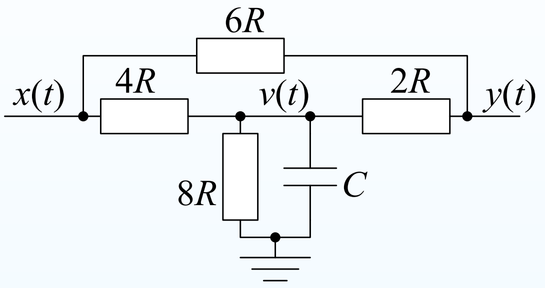
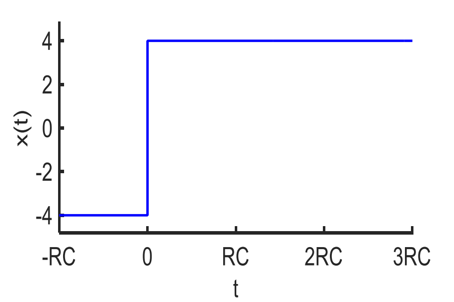
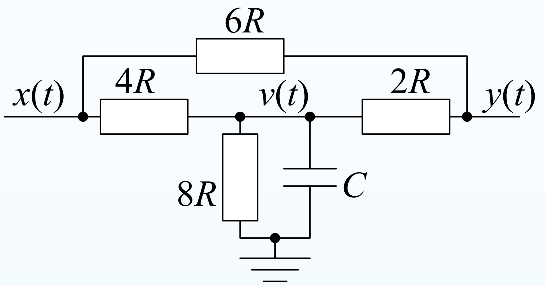
DC gain: $\frac{13}{16}$, HF gain: $\frac{8}{32} = \frac{1}{4}$, $\tau = \frac{32RC}{16} = 2RC$





Transient from Transfer Function



Calculate Transfer Function

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KCL @ Y: $\frac{Y-V}{2R} + \frac{Y-X}{6R} = 0$

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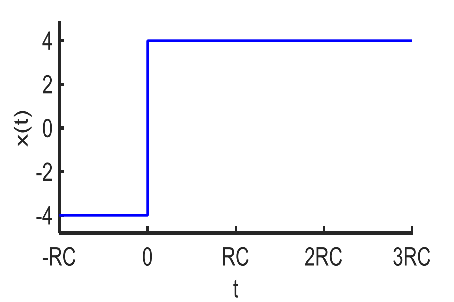
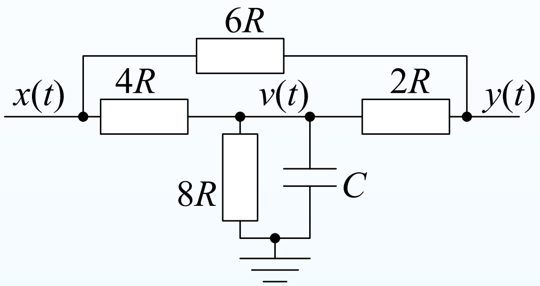


Steady State

Transient from Transfer Function



Calculate Transfer Function

KCL @ V: $\frac{V-X}{4R} + \frac{V}{8R} + j\omega CV + \frac{V-Y}{2R} = 0$

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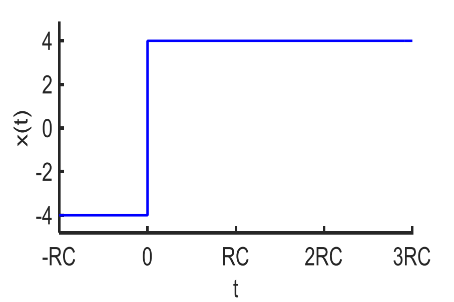
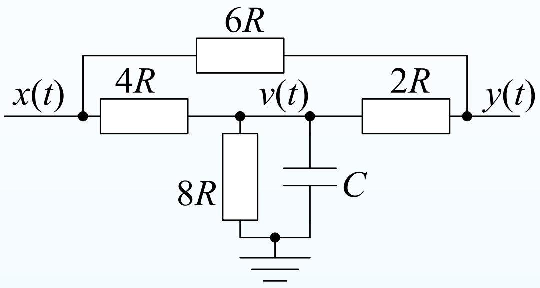


Steady State

$t < 0$: $y_{SS}(t) = \frac{13}{16}x(t) = \frac{13}{16} \times -4 = -3\frac{1}{4}$

Transient from Transfer Function



Calculate Transfer Function

KCL @ V: $\frac{V-X}{4R} + \frac{V}{8R} + j\omega CV + \frac{V-Y}{2R} = 0$

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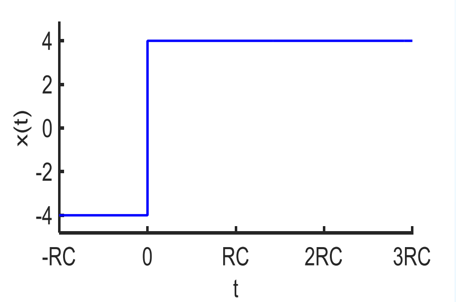

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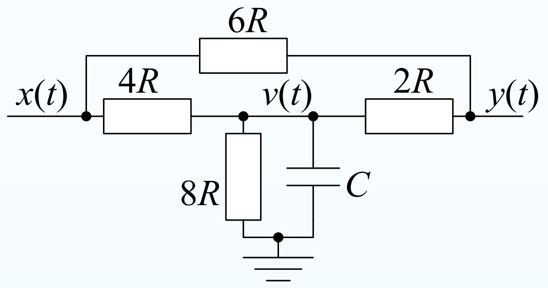
Transient from Transfer Function



Calculate Transfer Function

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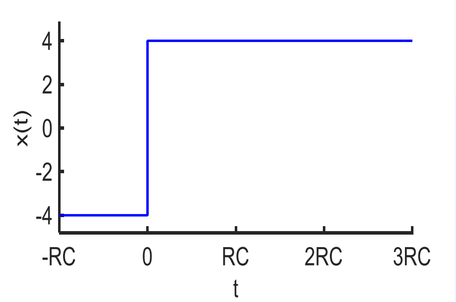

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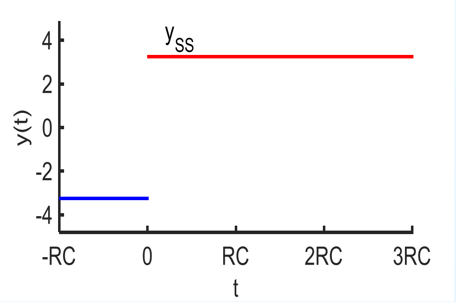
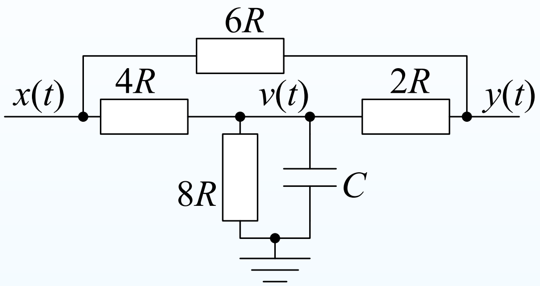


Transient from Transfer Function



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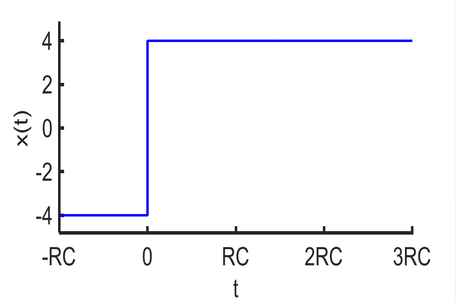

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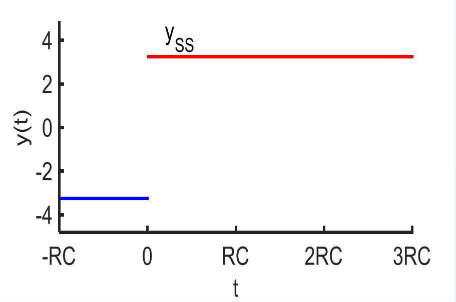

Steady State + Transient (for $t > 0$)

$t \geq 0$: $y = 3\frac{1}{4} + Ae^{-t/\tau}$

Transient from Transfer Function

Calculate Transfer Function

KCL @ V: $\frac{V-X}{4R} + \frac{V}{8R} + j\omega CV + \frac{V-Y}{2R} = 0$

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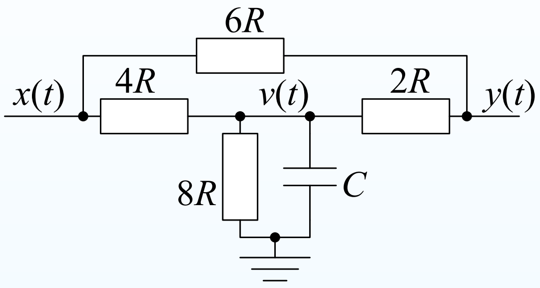

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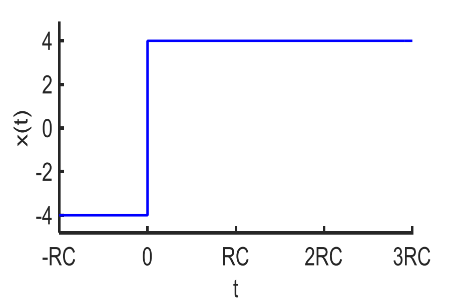

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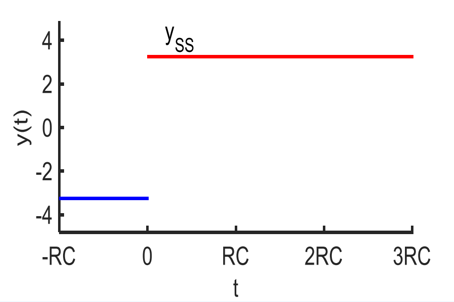
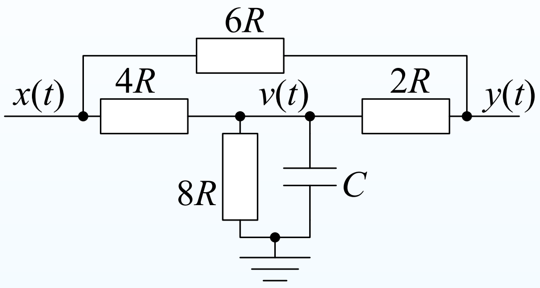
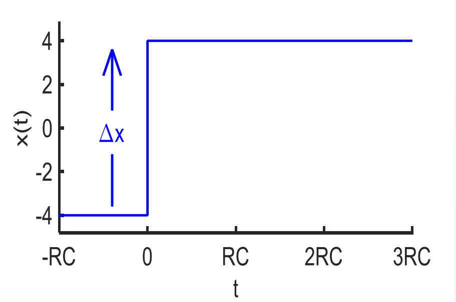
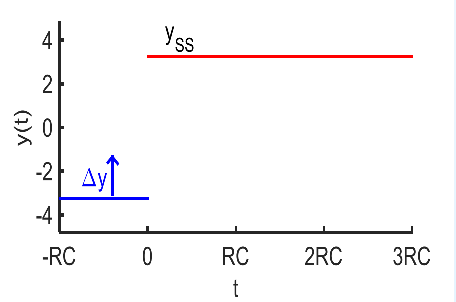


Discontinuity Gain (= HF Gain @ $\omega = \infty$)

Transient from Transfer Function







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DC gain: $\frac{13}{16}$, HF gain: $\frac{8}{32} = \frac{1}{4}$, $\tau = \frac{32RC}{16} = 2RC$

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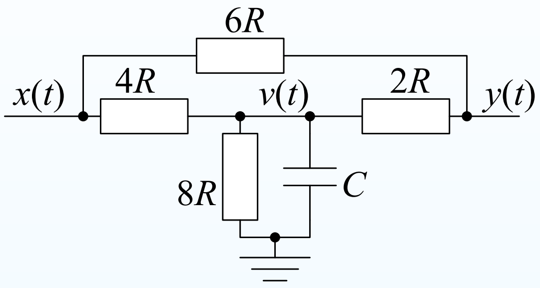
Steady State + Transient (for $t > 0$)

$t \geq 0$: $y = 3\frac{1}{4} + Ae^{-t/\tau}$

Discontinuity Gain (= HF Gain @ $\omega = \infty$)

$\Delta y = y(0+) - y(0-) = \frac{1}{4}\Delta x = \frac{1}{4} \times 8 = 2$

Transient from Transfer Function



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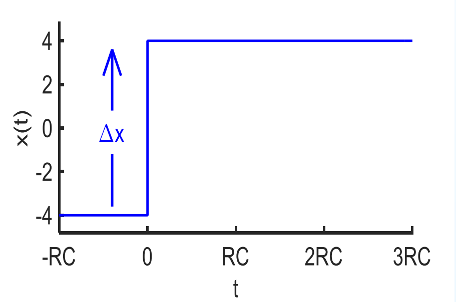

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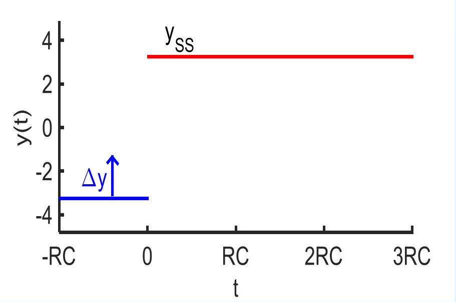

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Transient from Transfer Function

Calculate Transfer Function

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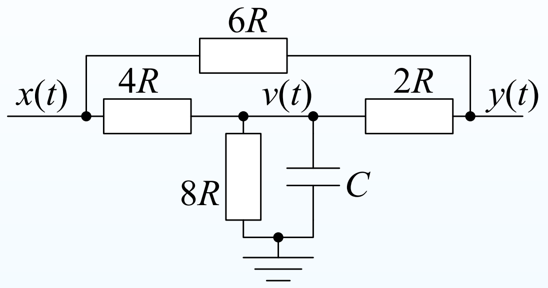

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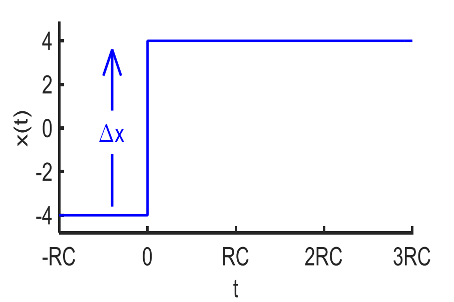

Steady State + Transient (for $t > 0$)

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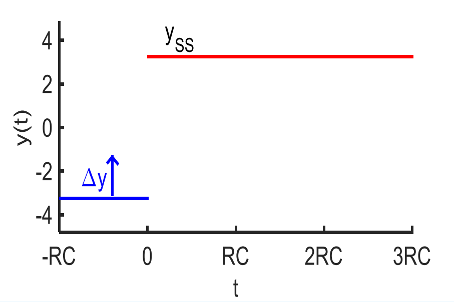

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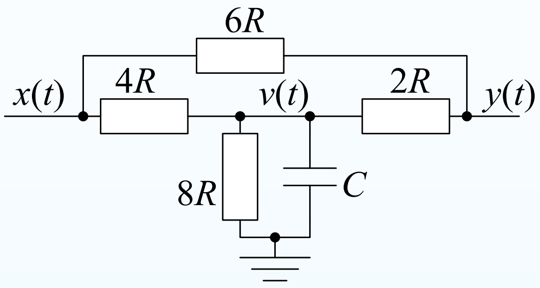
Transient from Transfer Function



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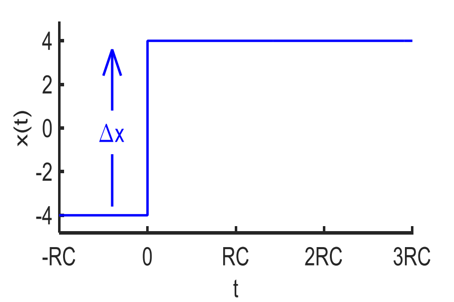

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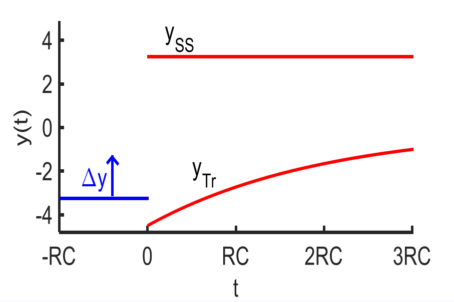
Steady State + Transient (for $t > 0$)

$t \geq 0$: $y = 3\frac{1}{4} + Ae^{-t/\tau}$



Discontinuity Gain (= HF Gain @ $\omega = \infty$)

$\Delta y = y(0+) - y(0-) = \frac{1}{4}\Delta x = \frac{1}{4} \times 8 = 2$

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Transient from Transfer Function

Calculate Transfer Function

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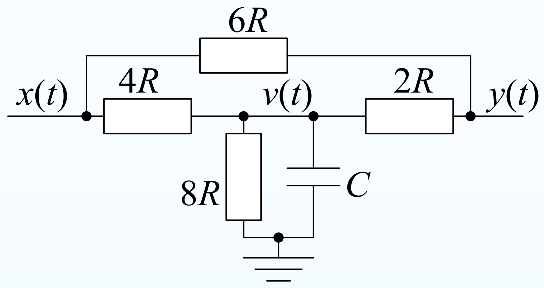

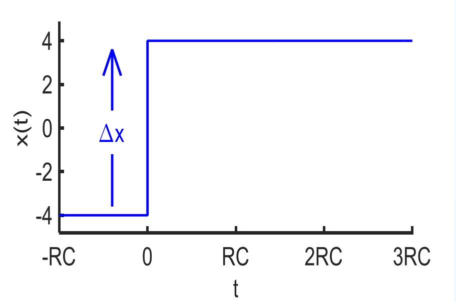

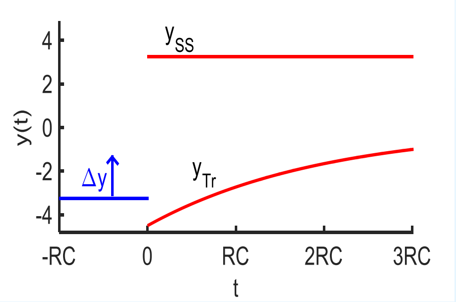

Steady State

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$\left(3\frac{1}{4} + A\right) - \left(-3\frac{1}{4}\right) = 2 \Rightarrow A = -4\frac{1}{2}$

Complete Expression

$t \geq 0$: $y(t) = 3\frac{1}{4} - 4\frac{1}{2}e^{-t/2RC}$

Transient from Transfer Function

Calculate Transfer Function

KCL @ V: $\frac{V-X}{4R}+\frac{V}{8R}+j\omega CV+\frac{V-Y}{2R}=0$

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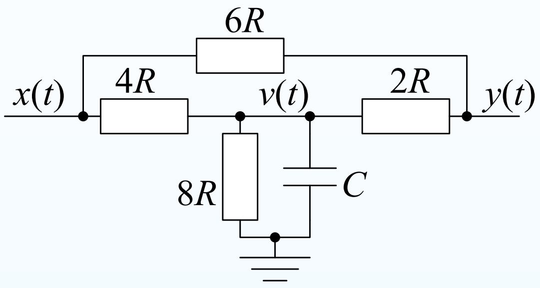

Steady State

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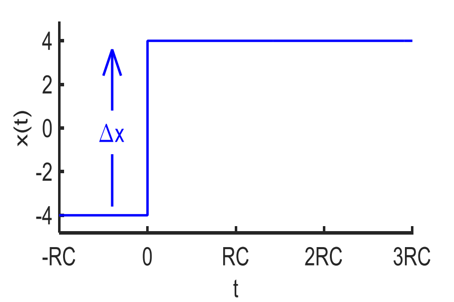

Steady State + Transient (for $t>0$)

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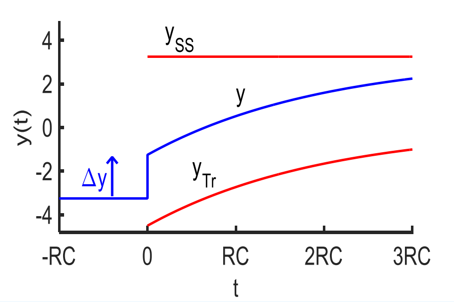

Discontinuity Gain (= HF Gain @ $\omega=\infty$)

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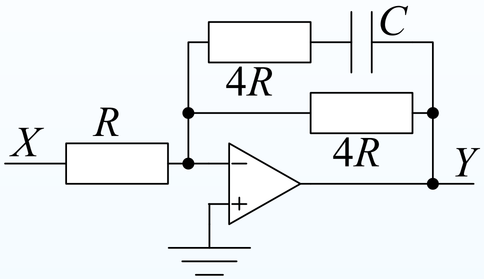
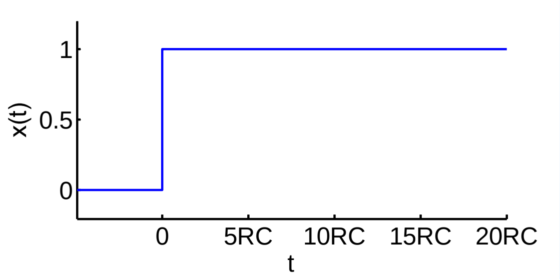
Complete Expression

$t\geq 0$: $y(t)=3\frac{1}{4}-4\frac{1}{2}e^{-t/2RC}$

Opamp Circuit Transient

Calculate Transfer Function (Inverting Amplifier)





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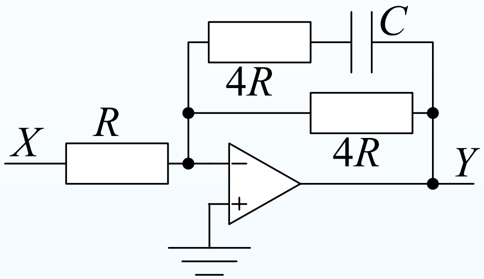

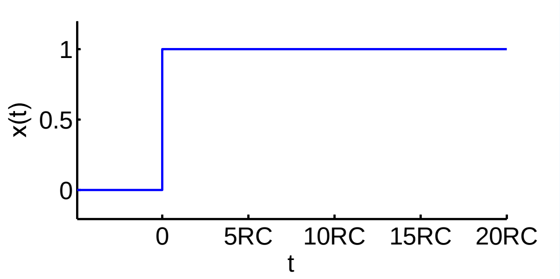

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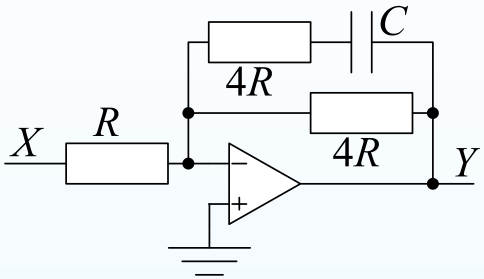

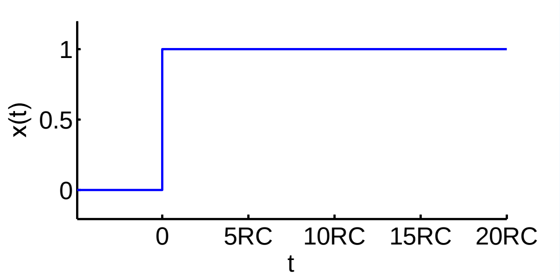

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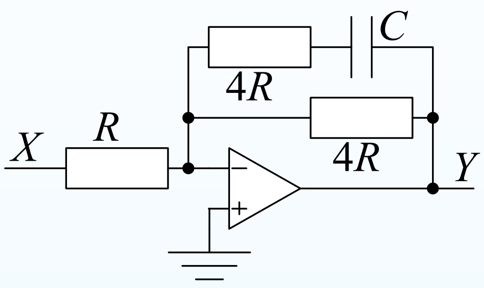

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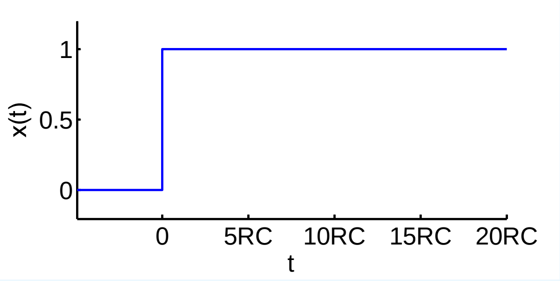

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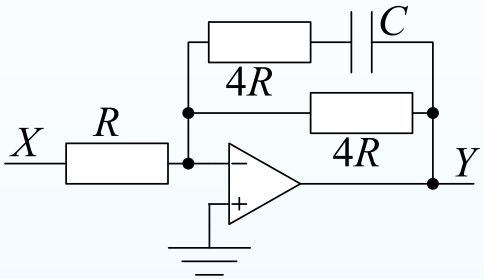

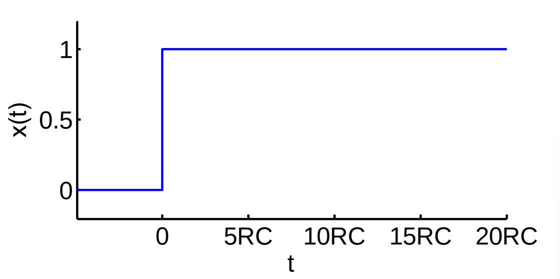

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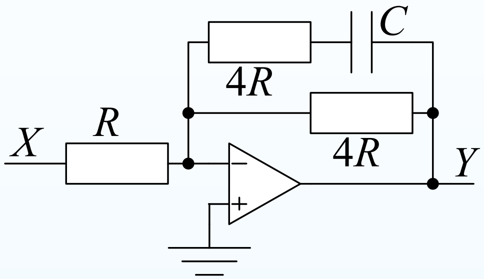

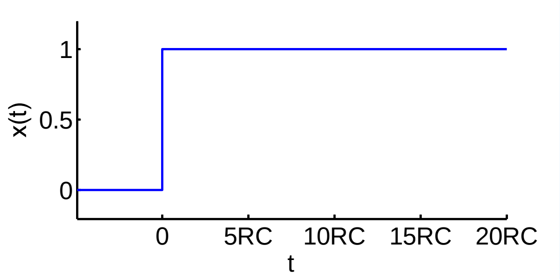

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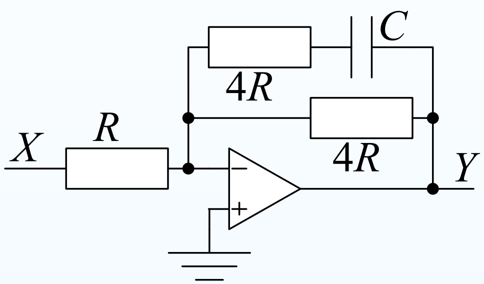

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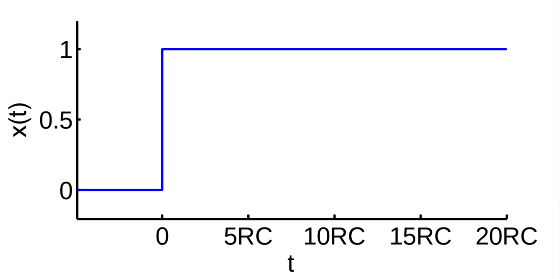

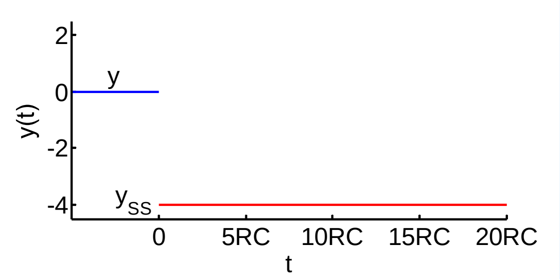

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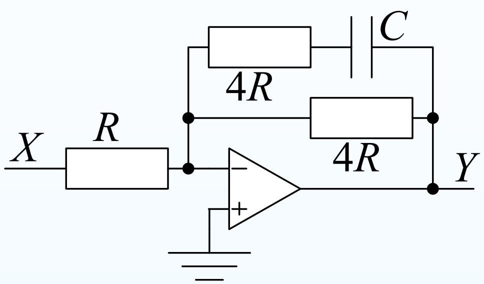

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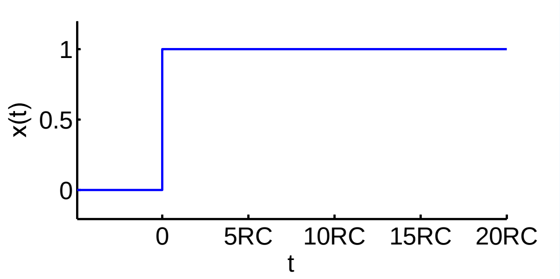

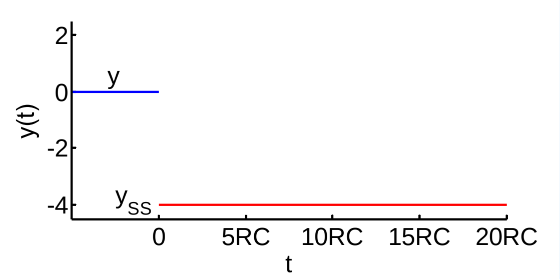

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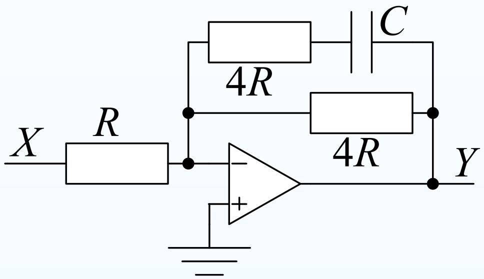

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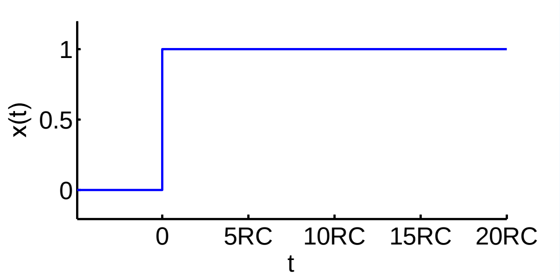

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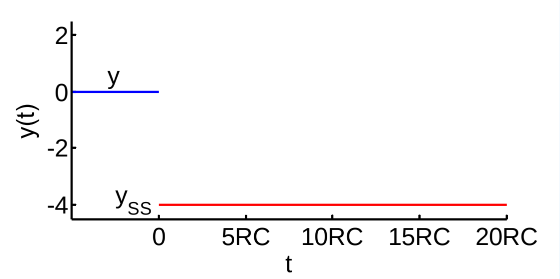

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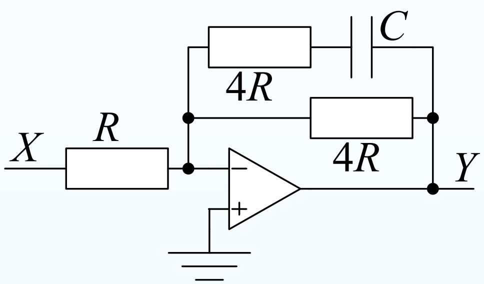

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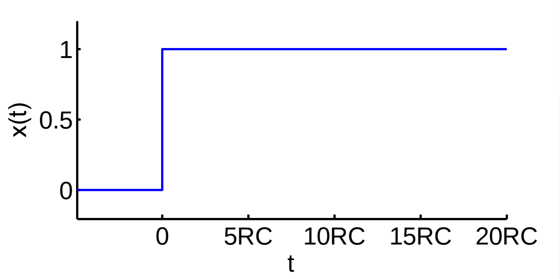

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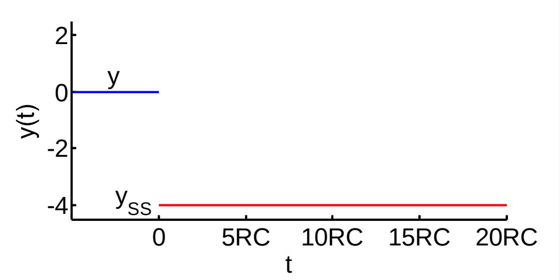

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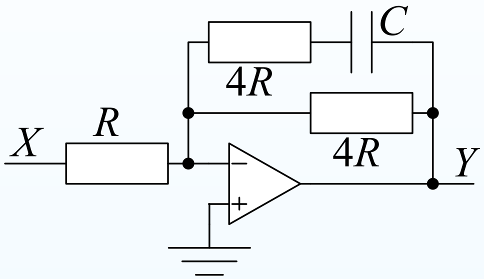

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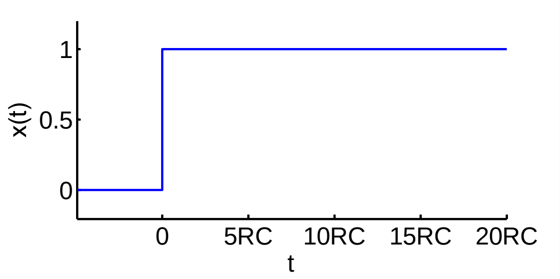

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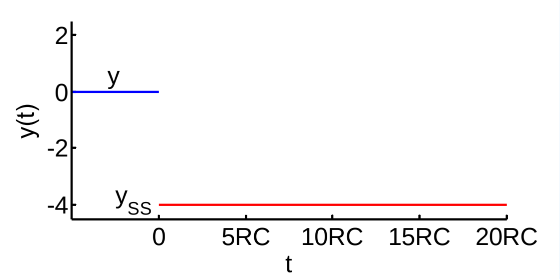

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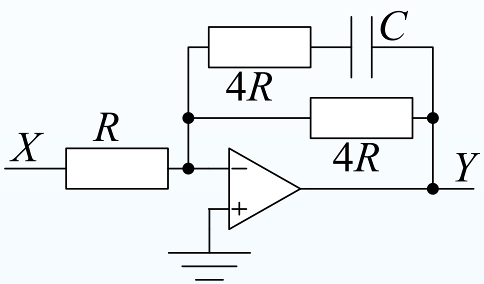

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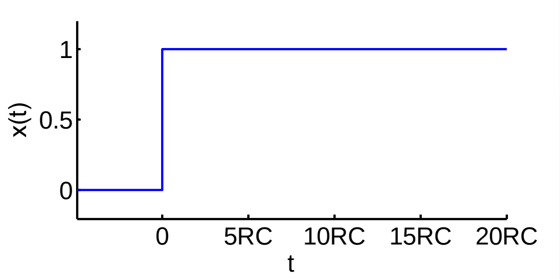

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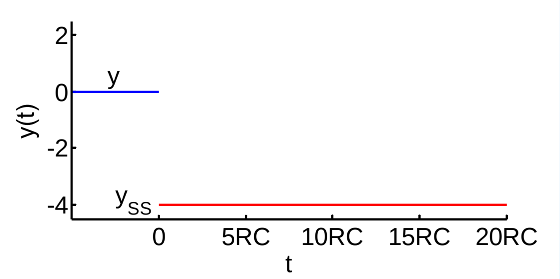

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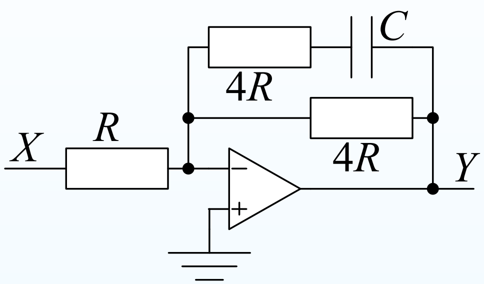

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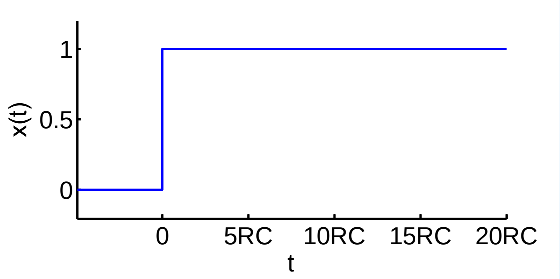

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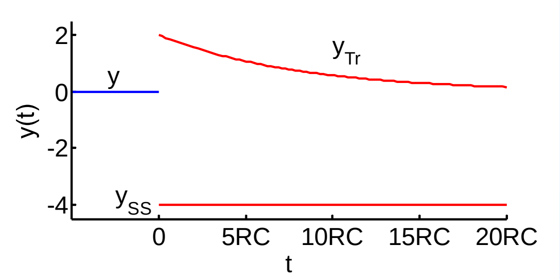

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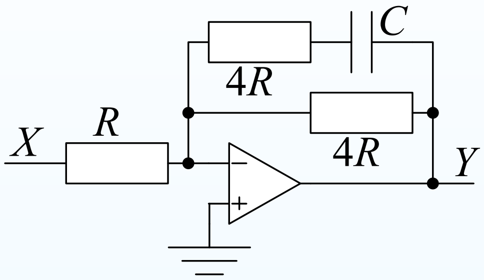

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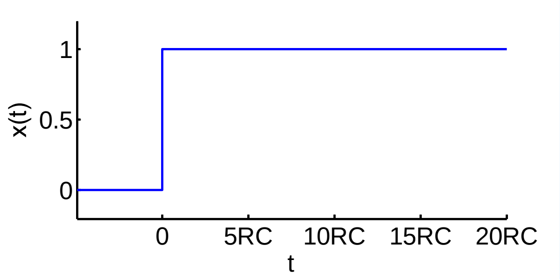

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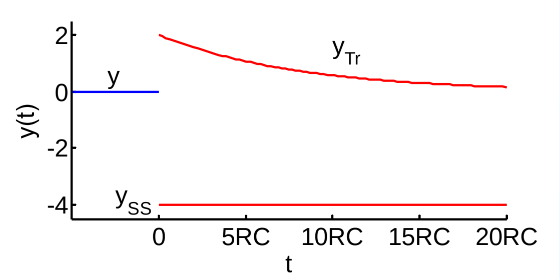

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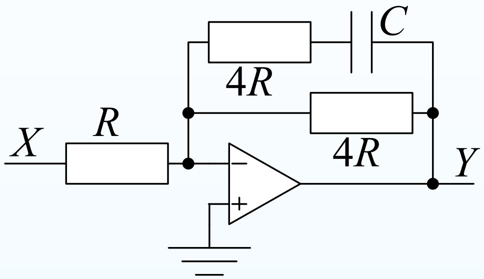

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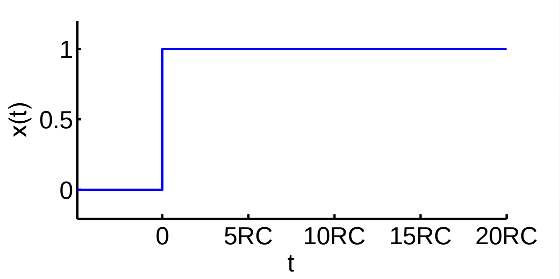

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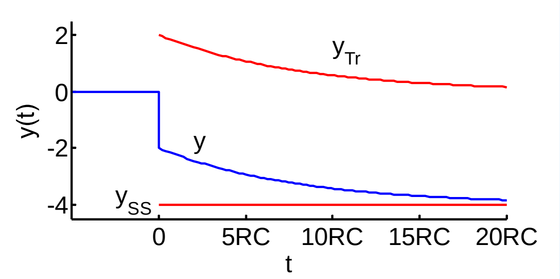

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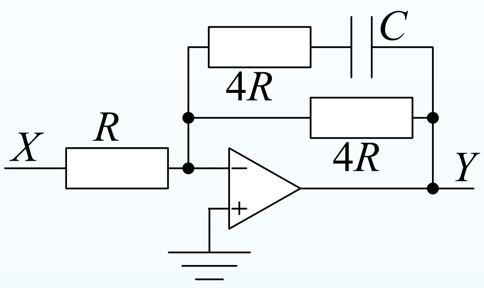

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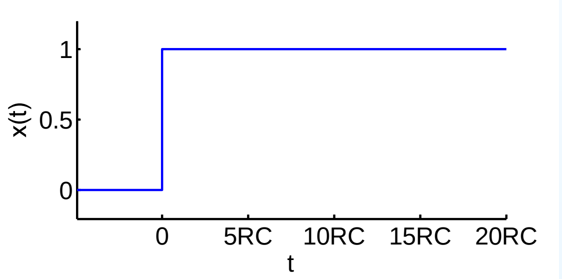

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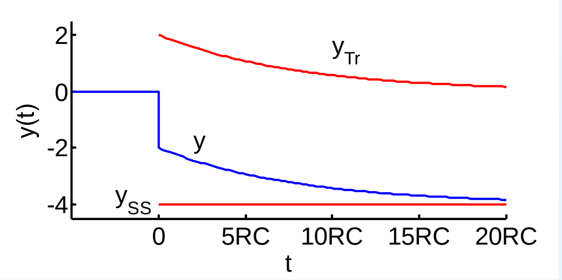

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For opamp circuits get τ from the transfer function because R_{Th} is difficult to work out.

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- Output is steady state + transient
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  - Two methods to find τ:
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For further details see Hayt Ch 8 or Irwin Ch 7.