▷ 16: Transients (B)
Piecewise steady state inputs
Sinusoidal Input
Multiple Discontinuities
Switched Circuit
Transfer Function
Transient from Transfer Function
Opamp Circuit Transient
Summary

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Summary

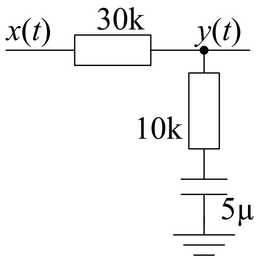
We will consider input signals that are sinusoidal or constant for a particular time interval and then suddenly change in amplitude, phase or frequency.

Output is the sum of the steady state and a transient:

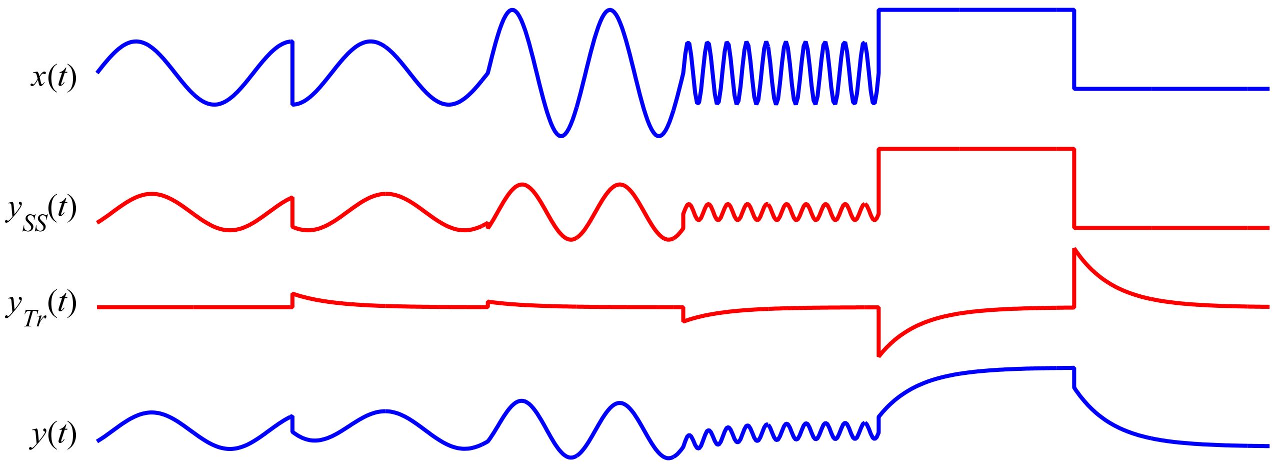
$$y(t) = y_{SS}(t) + y_{Tr}(t)$$

Steady state, $y_{SS}(t)$, is the same frequency as the input; use phasors + nodal analysis.

Transient is always $y_{Tr}(t) = Ae^{-\frac{t}{\tau}}$ at each change.



[only one C or L]



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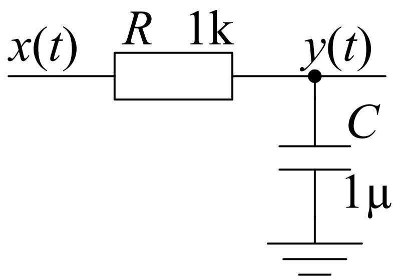
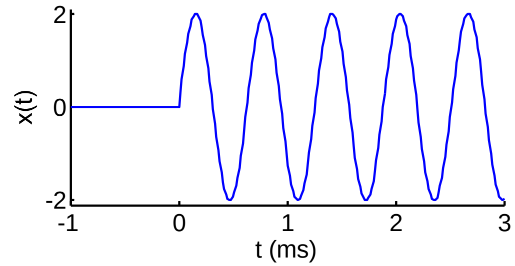
For $t < 0$: $y(t) = x(t) = 0$
For $t \geq 0$: $x = 2\sin\omega t \Rightarrow X = -2j$
$\tau = RC = 1\,\text{ms},\ \omega = 10\,\text{krad/s}$



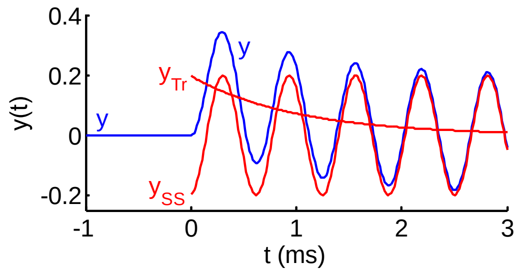
Steady State (for $t \geq 0$)

$$\frac{Y}{X} = \frac{1}{j\omega RC+1} = 0.1\angle-84^\circ$$
$$Y = X \times \frac{Y}{X} = -2j \times 0.1\angle-84^\circ$$
$$y_{SS}(t) = 0.2\cos(\omega t - 174^\circ)$$



Steady State + Transient

$$y(t) = 0.2\cos(\omega t - 174^\circ) + Ae^{-t/\tau}$$



Transient Amplitude

$$y(0+) = 0.2\cos(-174^\circ) + A$$
$$= -0.198 + A$$

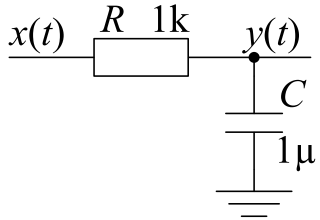
$$y(0+) = y(0-) = 0 \Rightarrow A = 0.198 \Rightarrow y_{Tr}(t) = 0.198e^{-t/\tau}$$

Complete Expression for $y(t)$

$$y(t) = 0.2\cos(\omega t - 174^\circ) + 0.198e^{-t/\tau}$$

Multiple Discontinuities

For $0 \le t < 0.2\pi$ ms: $X = -2j$, $\omega_1 = 10\,\text{k}$, $\tau = 1\,\text{ms}$
prev page $\Rightarrow y(t) = 0.2\cos(\omega t - 174°) + 0.198e^{-t/\tau}$



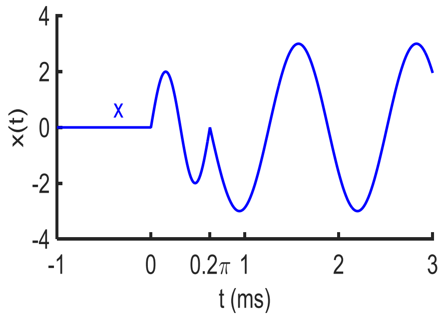
Steady State (for $t \ge 0.0002\pi = 0.63\,\text{ms}$)

$X = -3j$, $\omega_2 = 5\,\text{k}$

$\frac{Y}{X} = \frac{1}{j\omega_2 RC+1} = 0.2\angle -79°$

$Y = X \times \frac{Y}{X} = -3j \times 0.2\angle -79°$

$y_{SS}(t) = 0.59\cos(\omega_2 t - 169°)$

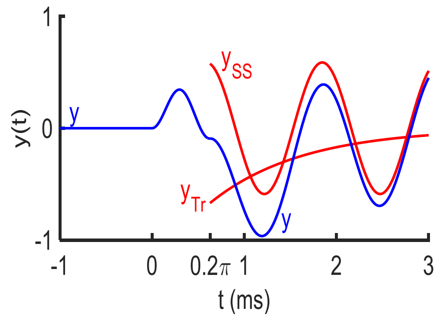


Steady State + Transient (for $t \ge 0.63\,\text{ms}$)

$y = 0.59\cos(\omega_2 t - 169°) + Be^{-(t-0.00063)/\tau}$

Transient Amplitude (at $t = 0.63\,\text{ms}$)

$y(0.00063+) = 0.59\cos(0.00063\omega_2 - 169°) + B$
$= 0.577 + B$



$y(0.00063-) = 0.2\cos(0.00063\omega_1 - 174°) + 0.198e^{-0.00063/\tau} = -0.092$

$\Rightarrow 0.577 + B = -0.092 \Rightarrow B = -0.67 \Rightarrow y_{Tr} = -0.67e^{-(t-0.00063)/\tau}$

Complete Expression for $y(t)$ (for $t \ge 0.63\,\text{ms}$)

$y(t) = 0.59\cos(\omega_2 t - 169°) - 0.67e^{-(t-0.00063)/\tau}$

Switched Circuit

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Operating the switch changes τ:

Closed: $\tau_C = (1\,\text{k}\,||\,9\,\text{k}) \times C = 0.9\,\text{ms}$

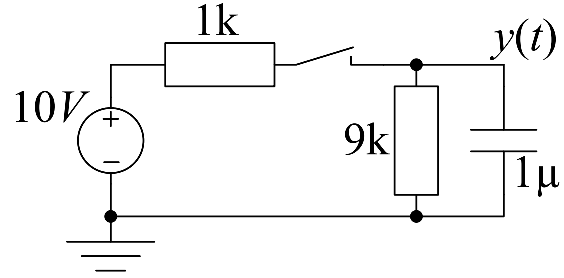
Open: $\tau_O = 9\,\text{k} \times C = 9\,\text{ms}$

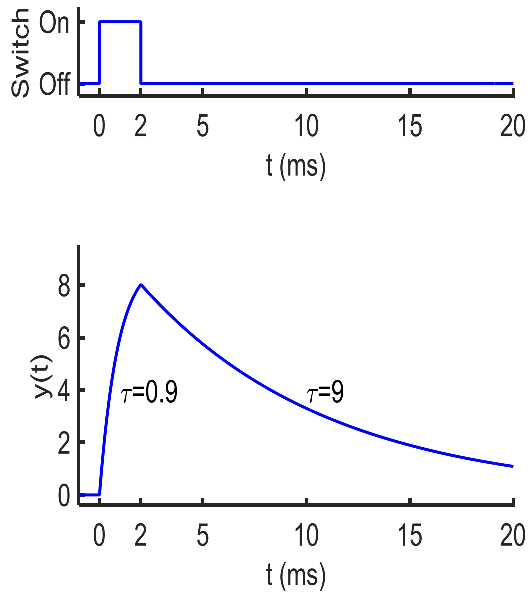
Switch closed at $t = 0$.

$$y_{SS} = 10 \times \frac{9}{10} = 9\,\text{V}$$
$$y(t) = 9 - 9e^{-t/\tau_C}$$
$$y(2-) = 9 - 9e^{-2/0.9} = 8.02$$

Switch opened at $t = 2$.

$$y_{SS} = 0\,\text{V}$$
$$y(t) = 0 + Ae^{-(t-2)/\tau_O}$$
$$y(2+) = A = y(2-) = 8.02$$
$$y(20) = 8.02e^{-(20-2)/9} = 1.09$$



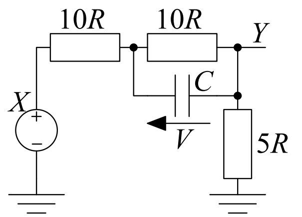


Transfer Function

Phasor nodal analysis:

$$\frac{Y}{X} = \frac{5R}{15R+\frac{10R}{1+10j\omega RC}} = \frac{10j\omega RC+1}{30j\omega RC+5} = 0.2\frac{\frac{j\omega}{p}+1}{\frac{j\omega}{q}+1}$$

Corner freqencies: $p = \frac{1}{10RC}$, $q = \frac{1}{6RC}$, HF gain $= \frac{1}{3}$

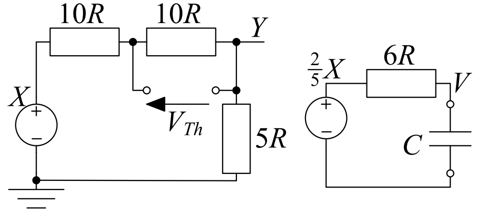


Thévenin Equivalent driving C:

$V_{Th} = \frac{2}{5}X$, $R_{Th} = 10R||15R = 6R$, $\tau = 6RC$

$V = \frac{2}{5}X \times \frac{1}{6j\omega RC+1} = \frac{2}{5}X \times \frac{1}{j\omega\tau+1}$

Denominator is always $(j\omega\tau + 1)$

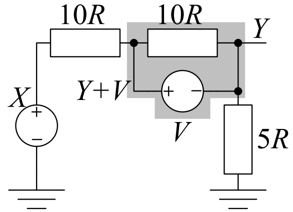


Linearity: $Y = aX + bV$

KCL @ supernode: $\frac{(Y+V)-X}{10R} + \frac{Y}{5R} = 0 \Rightarrow 3Y + V - X = 0$

$Y = \frac{1}{3}X - \frac{1}{3}V = \frac{1}{3}X - \frac{2}{15}X\left(\frac{1}{j\omega\tau+1}\right) = \frac{X}{15}\left(\frac{5j\omega\tau+3}{j\omega\tau+1}\right)$

Denominator of bV is unchanged by adding aX



(1) Denominator corner frequency is always $\frac{1}{\tau}$ for any transfer function in the circuit.

(2) $V = 0$ at $\omega = \infty$, so since $Y = aX + bV$, $a = \left.\frac{Y}{X}\right|_{\omega=\infty}$(= HF-gain)

V is never discontinuous so ΔY discontinuity = HF-gain$\times\Delta X$ discontinuity

Transient from Transfer Function

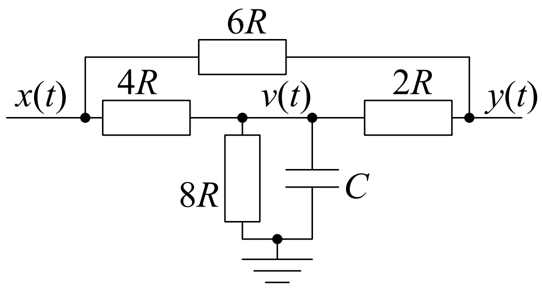
Calculate Transfer Function

KCL @ V: $\frac{V-X}{4R} + \frac{V}{8R} + j\omega CV + \frac{V-Y}{2R} = 0$

KCL @ Y: $\frac{Y-V}{2R} + \frac{Y-X}{6R} = 0$

$\rightarrow$ Transfer Function: $\frac{Y}{X} = \frac{8j\omega RC+13}{32j\omega RC+16}$

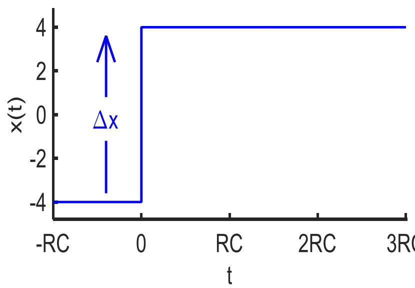
DC gain: $\frac{13}{16}$, HF gain: $\frac{8}{32} = \frac{1}{4}$, $\tau = \frac{32RC}{16} = 2RC$



Steady State

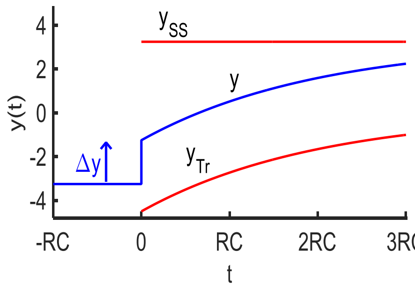
$t < 0$: $y_{SS}(t) = \frac{13}{16}x(t) = \frac{13}{16} \times -4 = -3\frac{1}{4}$

$t \geq 0$: $y_{SS}(t) = \frac{13}{16}x(t) = \frac{13}{16} \times +4 = +3\frac{1}{4}$



Steady State + Transient (for $t > 0$)

$t \geq 0$: $y = 3\frac{1}{4} + Ae^{-t/\tau}$



Discontinuity Gain (= HF Gain @ $\omega = \infty$)

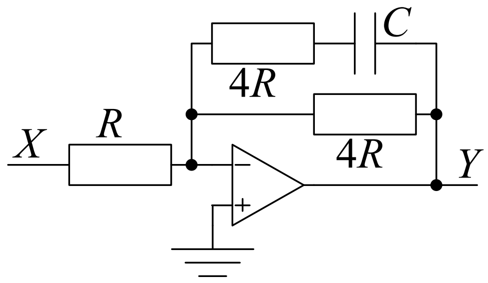
$\Delta y = y(0+) - y(0-) = \frac{1}{4}\Delta x = \frac{1}{4} \times 8 = 2$

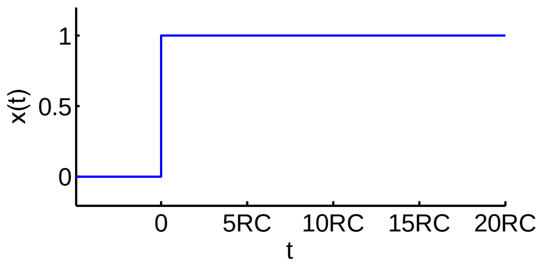
$\left(3\frac{1}{4} + A\right) - \left(-3\frac{1}{4}\right) = 2 \Rightarrow A = -4\frac{1}{2}$

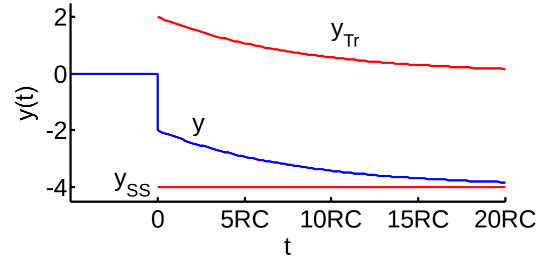
Complete Expression

$t \geq 0$: $y(t) = 3\frac{1}{4} - 4\frac{1}{2}e^{-t/2RC}$

Opamp Circuit Transient







Calculate Transfer Function (Inverting Amplifier)

$$\frac{Y}{X} = -\frac{Z_F}{R} = -\frac{1}{R} \times \frac{4R\left(4R+\frac{1}{j\omega C}\right)}{4R+\left(4R+\frac{1}{j\omega C}\right)} = -4\frac{4j\omega RC+1}{8j\omega RC+1}$$

DC gain: -4, HF gain: -2, $\tau = 8RC$

Steady State

$t < 0$: $y_{SS}(t) = -4v(t) = 0$

$t \geq 0$: $y_{SS}(t) = -4v(t) = -4 \times 1 = -4$

Steady State + Transient

$t \geq 0$: $y = -4 + Ae^{-t/\tau}$

Discontinuity Gain (= HF Gain)

$y(0+) - y(0-) = -2\left(x(0+) - x(0-)\right) = -2$

$(-4 + A) - (0) = -2 \Rightarrow A = 2$

Complete Expression

$t \geq 0$: $y(t) = -4 + 2e^{-t/8RC}$

For opamp circuits get τ from the transfer function because R_{Th} is difficult to work out.

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▷ Summary

- 1st order transients: circuits with only one C or L
- Transients arise from abrupt changes in the frequency, phase or amplitude of the input signal or else a switch changing
- Output is steady state + transient
- Steady state: nodal analysis $\rightarrow$ transfer function
- Transient: $Ae^{-t/\tau}$ where:
  - Two methods to find τ:
    - ▷ Thévenin seen by L or C: $\tau = R_{Th}C$ or $\frac{L}{R_{Th}}$
    - ▷ Transfer function denominator: $(aj\omega + b) \Rightarrow \tau = \frac{1}{\omega_c} = \frac{a}{b}$
  - Two methods to find A:
    - ▷ Continuity: $\Delta V_C = 0$ or $\Delta I_L = 0$
    - ▷ Discontinuity gain: Δoutput = HF gain $\times\Delta$input

For further details see Hayt Ch 8 or Irwin Ch 7.