E1.1 Circuit Analysis

Problem Sheet 1 - Solutions

1. Circuit (a) is a parallel circuit: there are only two nodes and all four components are connected between them.

   Circuit (b) is a series circuit: each node is connected to exactly two components and the same current must flow through each.

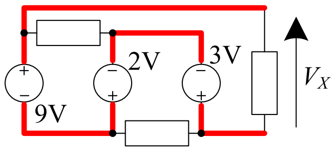
2. For subcircuit B the voltage and current correspond to the passive sign convention (i.e. the current arrow in the opposite direction to the voltage arrow) and so the power absorbed by B is given by $V \times I = 20\,\text{W}$.

   For device A we need to reverse the direction of the current to conform to the passive sign convention. Therefore the power absorbed by A is $V \times -I = -20\,\text{W}$.

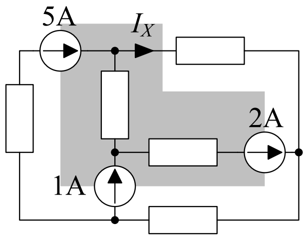
   As must always be true, the total power absorbed by all components is zero.

3. The power absorbed is positive if the voltage and current arrows go in opposite directions and negative if they go in the same direction. So we get: (a) $P_V = +4$, $P_I = -4$, (b) $P_V = +4$, $P_I = -4$, (c) $P_V = -4$, $P_I = +4$, (a) $P_V = -4$, $P_I = +4$. In all cases, the total power absorbed is $P_V + P_I = 0$.

4. We can find a path (shown highlighted below) from the bottom to the top of the V_X arrow that passes only through voltage sources and so we just add these up to get the total potential difference: $V_X = (-3) + (+2) + (+9) = +8\,\text{V}$.



5. If we add up the currents flowing out of the region shown highlighted below, we obtain $I_X - 5 - 1 + 2 = 0$. Hence $I_X = 4\,\text{A}$.



6. The three series resistors are equivalent to a single resistor with a value of $1 + 5 + 2 = 8\,\text{k}\Omega$.

7. The three series resistors are equivalent to a single resistor with a value of $\frac{1}{1/1+1/5+1/2} = \frac{1}{1.7} = 0.588\,\text{k}\Omega$.

8. We can first combine the parallel $2\,\text{k}$ and $3\,\text{k}$ resistors to give $\frac{2\times 3}{2+3} = 1.2\,\text{k}$. This is then in series with the $4\,\text{k}$ resistor which makes $5.2\,\text{k}$ in all. Now we just have three resistors in parallel to give a total of $\frac{1}{1/1+1/5+1/5.2} = \frac{1}{1.39} = 0.718\text{k}\Omega$.

9. The resistance is $\frac{1}{8\times 10^{-6}} = 125\,\text{k}\Omega$

10. [Method 1]: The resistors are in series and so form a potential divider. The total series resistance is $7\,\mathrm{k}$, so the voltages across the three resistors are $14 \times \frac{1}{7} = 2\,\mathrm{V}$, $14 \times \frac{2}{7} = 4\,\mathrm{V}$ and $14 \times \frac{4}{7} = 8\,\mathrm{V}$. The power dissipated in a resistor is $\frac{V^2}{R}$, so for the three resistors, this gives $\frac{2^2}{1} = 4\,\mathrm{mW}$, $\frac{4^2}{2} = 8\,\mathrm{mW}$ and $\frac{8^2}{4} = 16\,\mathrm{mW}$.

    [Method 2]: The total resistance is is $7\,\mathrm{k}$ so the current flowing in the circuit is $\frac{14}{7} = 2\,\mathrm{mA}$. The voltage across a resistor is IR which, in for these resistors, gives $2 \times 1 = 2\,\mathrm{V}$, $2 \times 2 = 4\,\mathrm{V}$ and $2 \times 4 = 8\,\mathrm{V}$. The power dissipated is VI which gives $2 \times 2 = 4\,\mathrm{mW}$, $4 \times 2 = 8\,\mathrm{mW}$ and $8 \times 2 = 16\,\mathrm{mW}$. The current through the voltage source is $2\,\mathrm{mA}$, so the power it is supplying is $VI = 14 \times 2 = 28\,\mathrm{mW}$. This is, inevitably, equal to the sum of the power disspipated by the three resistors: $4+8+16 = 28$.

11. [Method 1]: The resistors are in parallel and so form a current divider: the $21\,\mathrm{mA}$ will divide in proportion to the conductances: $1\,\mathrm{mS}$, $0.5\,\mathrm{mS}$ and $0.25\,\mathrm{mS}$. The total conductance is $1.75\,\mathrm{mS}$, so the three resistor currents are $21 \times \frac{1}{1.75} = 12\,\mathrm{mA}$, $21 \times \frac{0.5}{1.75} = 6\,\mathrm{mA}$ and $21 \times \frac{0.25}{1.75} = 3\,\mathrm{mA}$. The power dissipated in a resistor is I^2R which gives $12^2 \times 1 = 144\,\mathrm{mW}$, $6^2 \times 2 = 72\,\mathrm{mW}$ and $3^2 \times 4 = 36\,\mathrm{mW}$.

    [Method 2]: The equivalent resistance of the three resistors is $\frac{1}{1/1+1/2+1/4} = \frac{4}{7}\,\mathrm{k}\Omega$. Therefore the voltage across all components in the parallel circuit is $21 \times \frac{4}{7} = 12\,\mathrm{V}$. The current through a resistor is $\frac{V}{R}$ which gives $\frac{12}{1} = 12\,\mathrm{mA}, \frac{12}{2} = 6\,\mathrm{mA}$ and $\frac{12}{4} = 3\,\mathrm{mA}$. The power dissipated in a resistor is VI which gives $12 \times 12 = 144\,\mathrm{mW}$, $12 \times 6 = 72\,\mathrm{mW}$ and $12 \times 3 = 36\,\mathrm{mW}$. The power supplied by the current source is $12 \times 21 = 252\,\mathrm{mW}$ which as expected equals $144 + 72 + 36$.

12. The resistors form a potential divider, so $\frac{Y}{X} = \frac{4}{R_1+4}$. So we want $\frac{4}{R_1+4} = \frac{1}{4} \Rightarrow R_1 + 4 = 16 \Rightarrow R_1 = 12\,\mathrm{k}$.

13. The resistors form a potential divider, so $\frac{Y}{X} = \frac{R_2}{R_1+R_2}$. So we want $\frac{R_2}{R_1+R_2} = \frac{1}{10}$ and $R_1 + R_2 = 10\,\mathrm{M}\Omega$. Substituting one into the other and cross-multiplying gives $10R_2 = 10\,\mathrm{M}\Omega \Rightarrow R_2 = 1\,\mathrm{M}\Omega$. Substituting this into the simpler of the two initial equations gives $R_1 = 10 - 1 = 9\,\mathrm{M}\Omega$.

14. (a) $3\,\mathrm{k} = 1.5\,\mathrm{k} + 1.5\,\mathrm{k} = 3.3\,\mathrm{k}||33\,\mathrm{k}$, (b) $4\,\mathrm{k} = 3.9\,\mathrm{k} + 100$, (c) $3.488\,\mathrm{k} = 3.9\,\mathrm{k}||33\,\mathrm{k}$.

    To make an exhaustive search for creating a resistance of R, you need to consider two possibilities: (i) for two resistors in series, the largest of the two resistors must be in the range $[\frac{1}{2}R, R]$ or (ii) for two resistors in parallel, the smallest resistor must be in the range $[R, 2R]$. In both cases there are at most four possibilities, so you need to consider up to eight possibilities in all. So, for example, for $R = 3.5\,\mathrm{k}$, we would consider the following possibilities: (i) $1.5\,\mathrm{k} + 1.8\,\mathrm{k} = 3.3\,\mathrm{k}$, $1.8\,\mathrm{k} + 1.8\,\mathrm{k} = 3.6\,\mathrm{k}$, $2.2\,\mathrm{k} + 1.2\,\mathrm{k} = 3.4\,\mathrm{k}$, $2.7\,\mathrm{k} + 0.82\,\mathrm{k} = 3.52\,\mathrm{k}$ and (ii) $3.9\,\mathrm{k}||33\,\mathrm{k} = 3.488\,\mathrm{k}$, $4.7\,\mathrm{k}||15\,\mathrm{k} = 3.579\,\mathrm{k}$, $5.6\,\mathrm{k}||10\,\mathrm{k} = 3.59\,\mathrm{k}$, $6.8\,\mathrm{k}||6.8\,\mathrm{k} = 3.4\,\mathrm{k}$. The choice with least error is the one given above.

    Since we are interested in % errors, we need to consider the ratio between resistor values. The largest ratio between successive resistors is the series is $\frac{15}{12} = 1.25$ (this includes the wraparound ratio of $\frac{100}{82} = 1.22$). The worst-case percentage error will arise if our target resistance is the mean of these two values, 13.5. The percentage error in choosing either one is then $\frac{1.5}{13.5} = 11.1\%$.