SOFT DECISIONS FOR DQPSK DEMODULATION FOR THE VITERBI DECODING OF THE CONVOLUTIONAL CODES

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ABSTRACT

The conventional soft decision algorithm for DQPSK uses only the differential angle between consecutive DQPSK symbols. However it is possible to improve the accuracy of the soft decision bits by taking the amplitude information of the two DQPSK symbols into account. This paper introduces novel soft decision algorithms based on this approach which give a performance improvement compared to the conventional methods equivalent to an SNR gain of up to 5 dB.

1. INTRODUCTION

Digital cellular systems such as IS-136 as well as the broadcasting systems such as Eureka 147 DAB system employ a convolutional encoder followed by DQPSK modulation. In such systems, having optimal soft decisions rather than hard decisions at the DQPSK demodulator allow the Viterbi decoder to give superior bit error rate performance at the receiver. The idea of soft decisions is to provide the next stage of decoding with some reliability information for the input bits in addition to their probable values.

Most commonly, the soft or hard decoding of DQPSK is done using the phase differential of successive DQPSK symbols[1]. However, it is possible to improve the estimation of the soft bits if the amplitude information contained in successive symbols is also used.

The use of soft bits at the input to the Viterbi decoder of a convolutional encoder is well established in the literature. A general theory of soft decision decoding is given in [2] where the author provides a detailed study of the generation and the use of reliability information in convolutional decoders as a modified MAP soft decoding algorithm.

When the output from an inner decoder is used as the input to an outer decoder, using soft outputs from the inner decoder can substantially improve the overall performance [2]. Another example is the iterative decoding of turbo codes, where two parallel-concatenated recursive systematic convolutional codes (RSC) are used to achieve near Shannon limit performance for low SNR values [3,4].

It is possible to use a Viterbi algorithm based on a trellis structure similar to the one given in [5] to decode DQPSK into soft bits. Therefore it is also possible to use the algorithms developed for turbo decoding, such as the Soft Output Viterbi Algorithm (SOVA) [4], to calculate an approximate likelihood ratio, which is the soft output for the DQPSK demodulated bits.

The method given in [5] uses SOVA for iterative turbo decoding in a system comprise of DQPSK, viewed as the outer code, followed by a convolution encoder. In contrast to the SOVA, the symbol-by-symbol maximum a posteriori (MAP) algorithm [3,6,7] calculates the exact log likelihood for each bit and therefore gives improved BER performance with added complexity. Modified forms of MAP algorithm, such as the Max-Log MAP Algorithm and Log-MAP Algorithm [3] are also presented in the literature as possible ways of calculating the soft output bits. All these methods, however, require the entire set of DQPSK symbols to calculate the soft bits for each bit and the computational complexity of the algorithm increases with the length of the DQPSK symbol sequence.

Multiple Symbol Differential Detection (MSDD) [8-10] can improve the detection of differentially encoded bits, but the output of the MSDD is hard bits and therefore we do not have soft bits to be passed into the next decoder. Therefore, these algorithms are not well suited to concatenated systems.

Our objective is to find an optimal soft output algorithm for $\pi/4$-DQPSK, which uses only a consecutive pair of DQPSK symbols, in order to improve the performance of the Viterbi decoder. In order to achieve this, we use the amplitude and phase information as well as the signal and noise power values of the received DQPSK symbols to decode the "soft" bits.

This paper is organized as follows. In section 2, an optimal soft decision decoding algorithms for $\pi/4$-DQPSK is presented for different propagation environments. BER results obtained from using the new soft decision algorithm in a DAB receiver are presented in

Section 3, where they are compared with those of conventional methods. Finally, section 4 draws conclusions.

2. OPTIMAL SOFT DECISION FOR $\pi/4$-DQPSK

The soft output of the decoder represents the log likelihood ratio (LLR) of the probabilities of a bit being zero and one [7]. Therefore,

$$soft\_bit = \log\left(\frac{p}{1-p}\right) \tag{1}$$

where p is the probability of the bit being zero

In this section, the outline of the theoretical derivation of optimal soft decisions for a $\pi/4$-DQPSK modulation system is presented. Only the information given by a consecutive pair of $\pi/4$-DQPSK symbols and the signal and noise power of the signal are used to calculate the soft decisions.

We define $\mathbf{d}_{n-1}$ and $\mathbf{d}_n$ to be the faded received complex symbols (without noise). If we assume the channel is slowly varying with respect to the symbol duration and therefore there is no relative phase rotation due to the channel in successive DQPSK symbols, then

$$\mathbf{d}_n = \mathbf{w}^{k_n}\mathbf{d}_{n-1} \text{ with } k_n \in \{1,3,5,7\} \tag{2}$$

where $\mathbf{w} = e^{j\frac{\pi}{4}}$

Assuming the noise corrupting the real and imaginary parts of $\mathbf{d}_{n-1}$ and $\mathbf{d}_n$ are i.i.d. Gasussian with zero mean and variance σ_N^2, the PDF of the received signal values, $\mathbf{s}_{n-1}$ and $\mathbf{s}_n$, are given by,

$$f(\mathbf{s}_i \mid \mathbf{d}_i) = \frac{1}{2\pi\sigma_N{}^2} e^{-\left(\frac{1}{2\sigma_N{}^2}\left|\mathbf{s}_i - \mathbf{d}_i\right|^2\right)} \tag{3}$$

We need to calculate the a posteriori probability of k_n in order to calculate the soft bits,

$$\begin{aligned} p(k_n \mid \mathbf{s}_n, \mathbf{s}_{n-1}) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(k_n, \mathbf{d}_{n-1} \mid \mathbf{s}_n, \mathbf{s}_{n-1})\, dx_{\mathbf{d}_{n-1}}\, dy_{\mathbf{d}_{n-1}} \\ &= \frac{p(k_n)}{f(\mathbf{s}_n, \mathbf{s}_{n-1})} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\mathbf{s}_n, \mathbf{s}_{n-1} \mid k_n, \mathbf{d}_{n-1})\, f(\mathbf{d}_{n-1})\, dx_{\mathbf{d}_{n-1}}\, dy_{\mathbf{d}_{n-1}} \end{aligned} \tag{4}$$

where $x_{\mathbf{d}_{n-1}}$ and $y_{\mathbf{d}_{n-1}}$ are the real and imaginary parts of $\mathbf{d}_{n-1}$

Using (3), (4) and the independent noise assumption, and also assuming that the all values of k_n are equally likely, $p(k_n) = 1/4$,

$$\begin{aligned} p(k_n \mid \mathbf{s}_n, \mathbf{s}_{n-1}) &= \frac{1/4}{(2\pi)^2\, \sigma_N{}^4\, f(\mathbf{s}_n, \mathbf{s}_{n-1})} \times \\ &\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-\left(\frac{1}{2\sigma_N{}^2}\left(\left|\mathbf{s}_{n-1} - \mathbf{d}_{n-1}\right|^2 + \left|\mathbf{w}^{-k_n}\mathbf{s}_n - \mathbf{d}_{n-1}\right|^2\right)\right)} f(\mathbf{d}_{n-1})\, dx_{\mathbf{d}_{n-1}}\, dy_{\mathbf{d}_{n-1}} \end{aligned} \tag{5}$$

For the PDF, $f(\mathbf{d}_{n-1})$, of the signal without noise, we use one of two models those correspond to a Rayleigh fading and a time invariant (TI) channels respectively.

2.1. Rayleigh Fading Channel

For a Rayleigh fading channel, we assume $x_{\mathbf{d}_{n-1}}$ and $y_{\mathbf{d}_{n-1}}$ to be zero mean Gaussian with variance σ_S^2 giving,

$$f(\mathbf{d}_{n-1}) = \frac{1}{2\pi\sigma_S^2} e^{-\left(\frac{\left|\mathbf{d}_{n-1}\right|^2}{2\sigma_S^2}\right)} \tag{6}$$

Substituting (6) into (5), and simplifying further we obtain,

$$p(k_n \mid \mathbf{s}_n, \mathbf{s}_{n-1}) = Ke^{\left(\mathbf{w}^{-k_n}\mathbf{s}_n\mathbf{s}_{n-1}^* + \mathbf{w}^{k_n}\mathbf{s}_n^*\mathbf{s}_{n-1}\right)\Big/\left(2\sigma_N^2\left(2+SNR^{-1}\right)\right)} \tag{7}$$

where $SNR = \sigma_S^2 / \sigma_N^2$ and K is a constant.

Therefore using natural logarithm in equation (1), the soft decision for the first bit is,

$$b_1 = \ln\left(\frac{p(1 \mid \mathbf{s}_n, \mathbf{s}_{n-1}) + p(7 \mid \mathbf{s}_n, \mathbf{s}_{n-1})}{p(3 \mid \mathbf{s}_n, \mathbf{s}_{n-1}) + p(5 \mid \mathbf{s}_n, \mathbf{s}_{n-1})}\right) \tag{8}$$

A similar result may be obtained for b_2. Substituting for the probabilities, and simplifying further, we obtain,

$$b_1 = \frac{\sqrt{2}\,\mathrm{Re}\left(\mathbf{s}_n\mathbf{s}_{n-1}^*\right)}{\sigma_N^2\left(SNR^{-1}+2\right)},\quad b_2 = \frac{\sqrt{2}\,\mathrm{Im}\left(\mathbf{s}_n\mathbf{s}_{n-1}^*\right)}{\sigma_N^2\left(SNR^{-1}+2\right)} \tag{9}$$

We denote this as Soft Optimal 2 in the rest of the paper.

If σ_N^2 and SNR remains constant for all DQPSK symbols, (9) may be simplified to,

$$b_1 = \mathrm{Re}\left\{\mathbf{s}_n\mathbf{s}_{n-1}^*\right\},\ b_2 = \mathrm{Im}\left\{\mathbf{s}_n\mathbf{s}_{n-1}^*\right\} \tag{10}$$

We denote this as Soft Optimal 1 in the rest of the paper.

2.2. Time Invariant (Non-fading) Channel

Examples of time invariant channels include both AWGN channel and the frequency selective multipath channel without any Doppler effect. For these channels we can assume that the amplitude of DQPSK symbols remains constant over time. In the following section, the amplitude and the phase $\mathbf{d}_{n-1}$ are denoted by $A_{d_{n-1}}$ and $\phi_{d_{n-1}}$ respectively. The phase is assumed to be uniformly distributed between 0 and 2π. Therefore the desired PDF of $\mathbf{d}_{n-1}$ is,

$$f\left(\mathbf{d}_{n-1}\right)=\frac{1}{2\pi A_{d_{n-1}}}\delta\left(A_{d_{n-1}}-A\right) \qquad (11)$$

We called this the "ring PDF" below.

Substituting (11) in (5), and integrate by changing the variable $\mathbf{d}_{n-1}$ into polar coordinates and simplifying the result we obtain,

$$p\left(k_n \mid \mathbf{s}_n,\mathbf{s}_{n-1}\right)=2\pi K_1 I_0\left(AA_{n,k_n}\middle/\sigma_N^2\right) \qquad (12)$$

where $A_{n,k_n}=\left|\mathbf{s}_{n-1}+\mathbf{w}^{-k_n}\mathbf{s}_n\right|$, K_1 is a constant , and I_0 is the modified Bessel function of order zero.

Therefore using the equation (1), the first soft bit is,

$$b_1=\ln\left(\frac{I_0\left(AA_{n,1}/\sigma_N^2\right)+I_0\left(AA_{n,7}/\sigma_N^2\right)}{I_0\left(AA_{n,3}/\sigma_N^2\right)+I_0\left(AA_{n,5}/\sigma_N^2\right)}\right) \qquad (13)$$

Similarly, the soft decision for the second bit,

$$b_2=\ln\left(\frac{I_0\left(AA_{n,1}/\sigma_N^2\right)+I_0\left(AA_{n,3}/\sigma_N^2\right)}{I_0\left(AA_{n,5}/\sigma_N^2\right)+I_0\left(AA_{n,7}/\sigma_N^2\right)}\right) \qquad (14)$$

We denote this as Soft Optimal 3 in the rest of the paper.

3. SIMULATION RESULTS

We have compared the performance of the optimal soft decision methods presented above with that of a conventional soft decision method, where the soft bits are represented by the sine and the cosine of the differential phase between consecutive DQPSK symbols. Simulations are done for AWGN and different fading channel models complying with the COST 207 [11] propagation environments. The signal is modeled according to the main service channel (MSC) specified in the DAB standards [12] assuming Transmission Mode 3, protection level 3 of equal error protection (EEP) set A, and L band (i.e. carrier frequency = 1.471GHz). σ_N^2 and SNR values are estimated from the received signal assuming the same set of values for one DAB frame.

Figure 1 shows the BER results as a function of the overall SNR for a non-fading, frequency non-selective propagation channel, with added coloured Gaussian noise (ACGN) having a hamming window shaped spectrum. The BER improvement arising for the optimal soft decisions corresponds to an SNR gain of around 1dB for low SNR values. Soft Optimal 3, derived for non-fading channel models has slightly better performance than Soft Optimal 2, which is optimal for Rayleigh channel models.

BER results for the rural-area fading channel (RA130)[11] with ACGN are given in Figure 2. There is a significant performance improvement using the optimal methods soft 1, 2 and 3 compared to conventional soft decision method corresponding to a SNR gain of about 5dB. Furthermore, the more complex soft decision method, Optimal 2, gives a considerable performance advantage over the simple soft decision method Optimal 1, which was deduced from Optimal 2.

BER results for typical urban propagation environment (TU15)[11] with ACGN are shown in the Figure 3. Optimal soft decision methods again offer a significant performance advantage over the conventional method corresponding to a SNR gain of 3 to 4 dB. Furthermore, Soft Optimal 2 gives about 1dB improvement over Soft Optimal 1.

Despite the slow time varying nature of the TU15 propagation environment we do not get better performance using Soft Optimal 3 compared to Soft Optimal 2. The reason for this is that the statistical properties of TU15, as shown in Figure 4 and Figure 5, are closer to that of the Gaussian distribution than to the ring distribution even though the fading is slow.

4. CONCLUSIONS

The use of an optimal soft decision algorithm compared to the conventional soft decision algorithm gives a performance improvement equivalent to an SNR gain of up to 5dB. In a time invariant propagation environment, Soft Optimal 3 gives the best performance whereas in time selective environment, Soft Optimal 2 gives the best performance. It is possible to switch between these two soft decision modes to obtain the best performance in a real receiver depending on the current status of the time selectivity of the received signal.

5. REFERENCES

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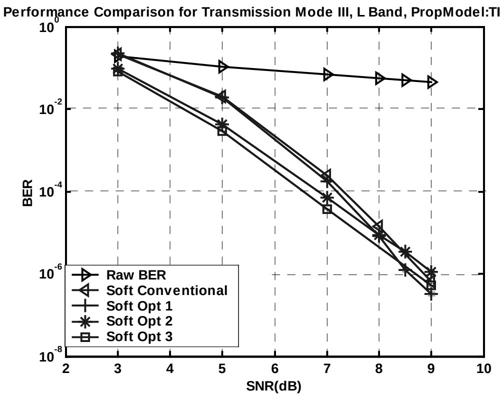

Figure 1: BER for MSC for Non Fading Channel with ACGN

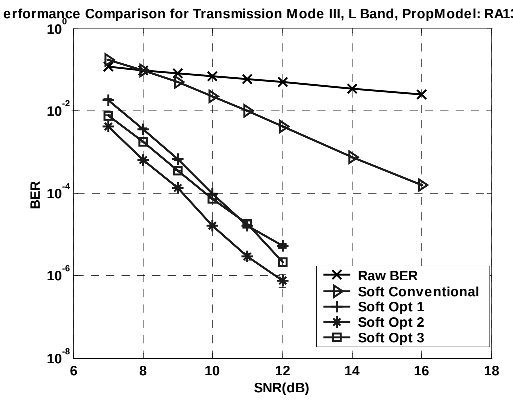

Figure 2: BER for MSC for Fading Channel RA130 with ACGN

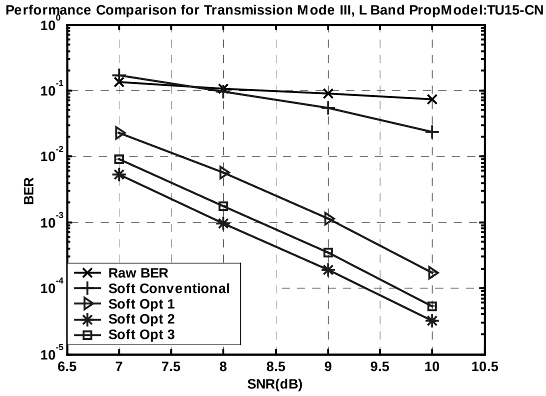

Figure 3: BER for MSC for Fading Channel TU15 with ACGN

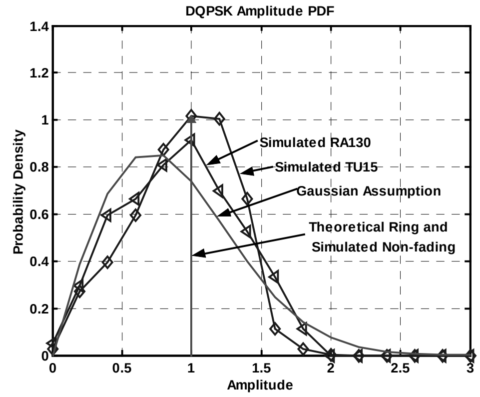

Figure 4: Statistical Properties of the Amplitude of DQPSK symbols

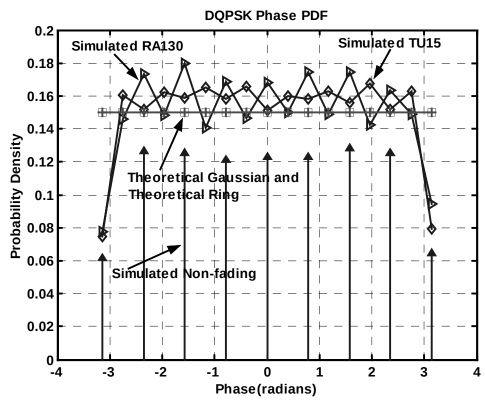

Figure 5: Statistical Properties of the Phase of DQPSK symbols