

# Bessel panels - high-power speaker systems with radial sound distribution

Bessel panels are simple loudspeaker systems which can produce radially-distributed sound with standard, low-cost speakers. They can be constructed without active or passive components, and will find extensive use in cinemas, theatres, and public-address systems.

Conventional high-power (200 W and above) speaker systems use either a single large speaker or a speaker array. However, both of these types of system have severe disadvantages.

Large speakers are extremely expensive, and the wide diaphragm movements needed to produce high sound levels can lead to severe distortion, noticeable in some cinemas which use single-speaker systems.

Speaker arrays on the other hand tend to concentrate the sound into a beam; the higher the frequency, the thinner the beam. This leads to poor sound quality for that part of the audience which is outside the beam. The more speakers in the array, the more pronounced is this effect.

It is already known that radially-distributed sound can be obtained from a row of speakers by making use of Bessel coefficients. The input to speaker m has a weighting factor  $J_{\rm m}(x)$ , i.e. the Bessel function of order m. Unfortunately these weighting factors are fractions, and such a system requires a complicated analogue or digital circuit. For this reason, the system has never been commercially feasible.

# SIMPLE-TO-IMPLEMENT BESSEL PANELS

Work done at the Philips Research Laboratories has shown that the complications can be eliminated by choosing a weighting factor of 1 for the outer speakers in the row, and higher weighting factors for the inner speakers. For five speakers, the weighting factors are:

$$A:B:C:D:E = 1:2n:2n^2:-2n:1$$

By choosing n=1, this is simplified to:

$$A:B:C:D:E = 1:2:2:-2:1$$

Each speaker is separated from its neighbour by a distance d1; the speaker layout is shown in Fig.1. The speakers can



Fig.1 Layout for five speakers

be connected in either of two ways, as shown in Fig.2. Standard 8  $\Omega$  speakers can be used.

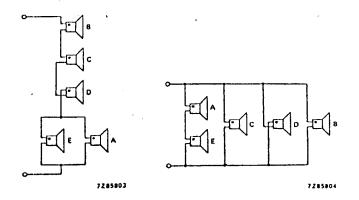


Fig.2 Circuit diagram for five speakers

In both of the circuits shown in Fig.2, speakers E and A (weighting factor of 1) draw half the current of B, C and D (weighting factor of 2). The negative weighting factor for D is obtained by reversing the speaker polarity. Which circuit is chosen depends only on the total impedance desired.

These systems are simple to construct, and need no additional components. They produce a sound distribution which is almost identical to that of a single speaker.

For a seven-unit system the weighting factors are:

A:B:C:D:E:F:G=1:2n:2
$$n^2$$
: $n^3-n$ : $-2n^2$ :2n: $-1$ 

With n = 1, this gives:

$$A:B:C:D:E:F:G = 1:2:2:0:-2:2:-1$$

where the weighting factor 0 means that speaker D can be omitted, so only six are actually required; the corresponding layout is shown in Fig.3. Two circuit diagrams are again possible, as shown in Fig.4.

The weighting factors for nine units are:

$$1:2n:2n^2:n^3-n:\frac{(n^4-1)}{4}-2n^2:-(n^3-n):2n^2:-2n:1$$

Fig.3 Layout for six speakers (seven weighting factors)

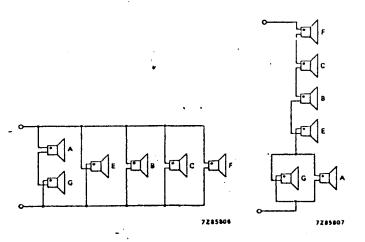


Fig.4 Circuit diagram for six speakers

With n = 1, this gives:

The speaker layouts for this system are similar to those shown in Figs.1 and 3; the circuit diagrams are similar to Figs.2 and 4.

# COMBINATION OF BESSEL PANELS

Bessel panels can also be combined to give different sound distributions in the vertical plane. A simple combination is shown in Fig.5; the number in each speaker shows the weighting factor. This will give a radial distribution of sound in the horizontal direction; the vertical distribution will be concentrated at the level of the panels.

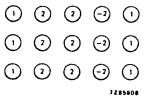


Fig.5 Three Bessel panels, arranged for vertical concentration of sound at level of panels

The more rows of speakers which are connected in this way, the more pronounced will be sound concentration at the level of the speakers; this system is suitable for a cinema or theatre where the sound must be concentrated at audience level.

The system in Fig.6, on the other hand, gives a hemispherical sound distribution because both the rows and the columns are connected according to the Bessel function. This system can easily be constructed with  $4\Omega$  and  $8\Omega$  speakers.

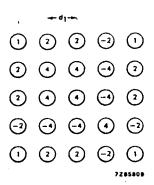


Fig.6 Arrangement for hemispherical sound distribution

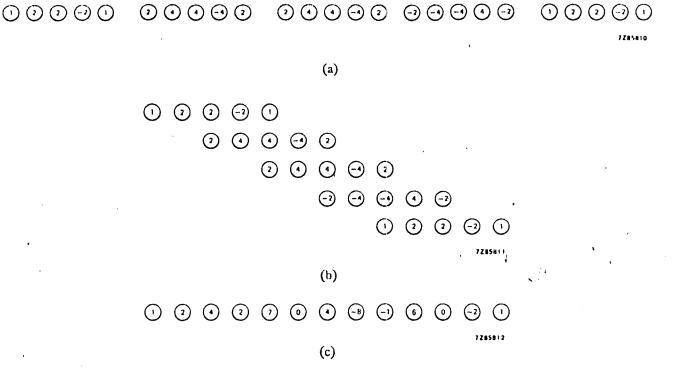


Fig.7 Linear combination of Bessel panels

Another possible combination is shown in Fig.7, which shows a row of five Bessel panels; Fig.7 (b) shows the same panels overlapping; Fig.7 (c) shows a single row of panels with the same output as Fig.7 (b), the weighting factors being obtained simply by adding the weighting factors in each column.

Figures 5 to 7 show some examples of the many 5-unit Bessel panel combinations which are possible. Similar arrangements are also possible with 7 or 9 units. The system chosen depends on the power output and sound distribution desired, and the output impedance of the driving amplifier(s). Weighting factors of units in the system are usually chosen to be as close to each other as possible, to ensure comparable loading of the speakers.

There is clearly a huge number of possible combinations, especially because each of the speakers shown in Figs. 5 to 7 may itself be replaced by either a multiway speaker system (woofer/tweeter), or a 5,7 or 9-unit Bessel panel.

One example of a system made up of separate Bessel panels is shown in Fig.8. Each square represents a Bessel panel made up of 5,7 or 9 speakers.

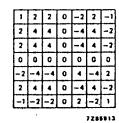


Fig.8 Combination of Bessel panels

#### STEREO BESSEL PANELS

Fig.9 shows the circuit diagram and the layout of a fiveunit Bessel panel similar to the mono panel shown in Fig.1. The weighting factors for this stereo Bessel panel are: A:B:C:D:E = 0.5(L+R):(L-R):(L+R):(R-L):0.5(L+R).

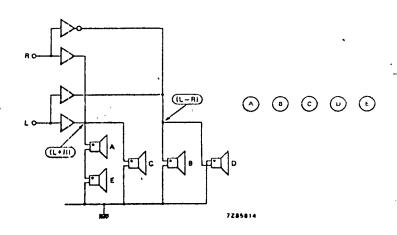


Fig.9 Stereo Bessel panel

Similar systems are of course possible with 7 or 9 speakers, or indeed with combinations of Bessel panels. Expanded stereo (in which the apparent stereo image is expanded) is also possible with Bessel panels. The circuit diagram for a stereo-base widening system is similar to that

shown in Fig.9, but signals (L+R) and (L-R) are taken to a balanace control. The outputs from the balance control are k(L+R), which is connected to speakers A, E and C, and (1-k) (L-R), which is taken to speakers B and D. For:

k = 1	mono
k = 0.5	stereo (and the configuration is that shown in Fig.9)
0 < k < 0.5	expanded stereo (sometimes called spatial stereo)
k = 0	super stereo.

# MICROPHONES CONNECTED ACCORDING TO BESSEL FUNCTION

Although this publication has described only Bessel loudspeaker panels, microphones can also be arrayed and connected according to the Bessel coefficients to give a system which has very high sensitivity, and which can give omnidirectional reproduction. The theory and implementation is exactly as given above.

### **BESSEL FUNCTIONS**

Friedrich Wilhelm Bessel (1784-1846) was a German mathematician and astronomer. He systematized Bessel functions in his investigations into heliocentricity (the theory that the sun is the centre of the solar system).

Bessel functions are for example used in the analysis of the vibrations of a stretched membrane, the transport of heat in a solid cylinder, the flow of electromagnetic waves along wires, the diffraction of light, the theory of elasticity, and in hydrodynamics.

The Bessel function  $J_{\hat{m}}$  of order m satisfies the differential equations:

$$\frac{\mathrm{d}^2 u}{\mathrm{d} p^2} + \frac{1}{p} \cdot \frac{\mathrm{d} u}{\mathrm{d} p} + \left(1 - \frac{m^2}{p^2}\right) u = 0$$

Bessel functions resemble decaying sines and cosines but are somewhat more complicated. They may be expressed as the series

$$J_{m}(p) = \left(\frac{p}{2}\right)^{m} \sum_{k=0}^{k=\infty} \left(\frac{-\frac{p^{2}}{4}}{k!(m+k)}\right)^{k}$$

Because Bessel functions have the property that:

$$|\sum_{m=-\infty}^{m=\infty} J_m(p) \exp(jmx)| = |\exp(jp \sin x)| = 1,$$

an infinite array of loudspeakers with individual input signals m weighted with  $J_{\rm m}(p)$  has the same directivity pattern as a single loudspeaker. If the parameter p is chosen to be small enough, the same is approximately true for a finite array. For a five speaker array, for example, it can be shown that choosing p to be approximately 1.5 gives values for the five coefficients of  $J_{\rm m}(1.5)$  to be approximately 1:2:2:-2:1.